Monte Carlo Simulation of NYMEX Crude Oil Option Prices Under A Generalized Hyperbolic Distribution

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What's Wrong with Normal Distribution?

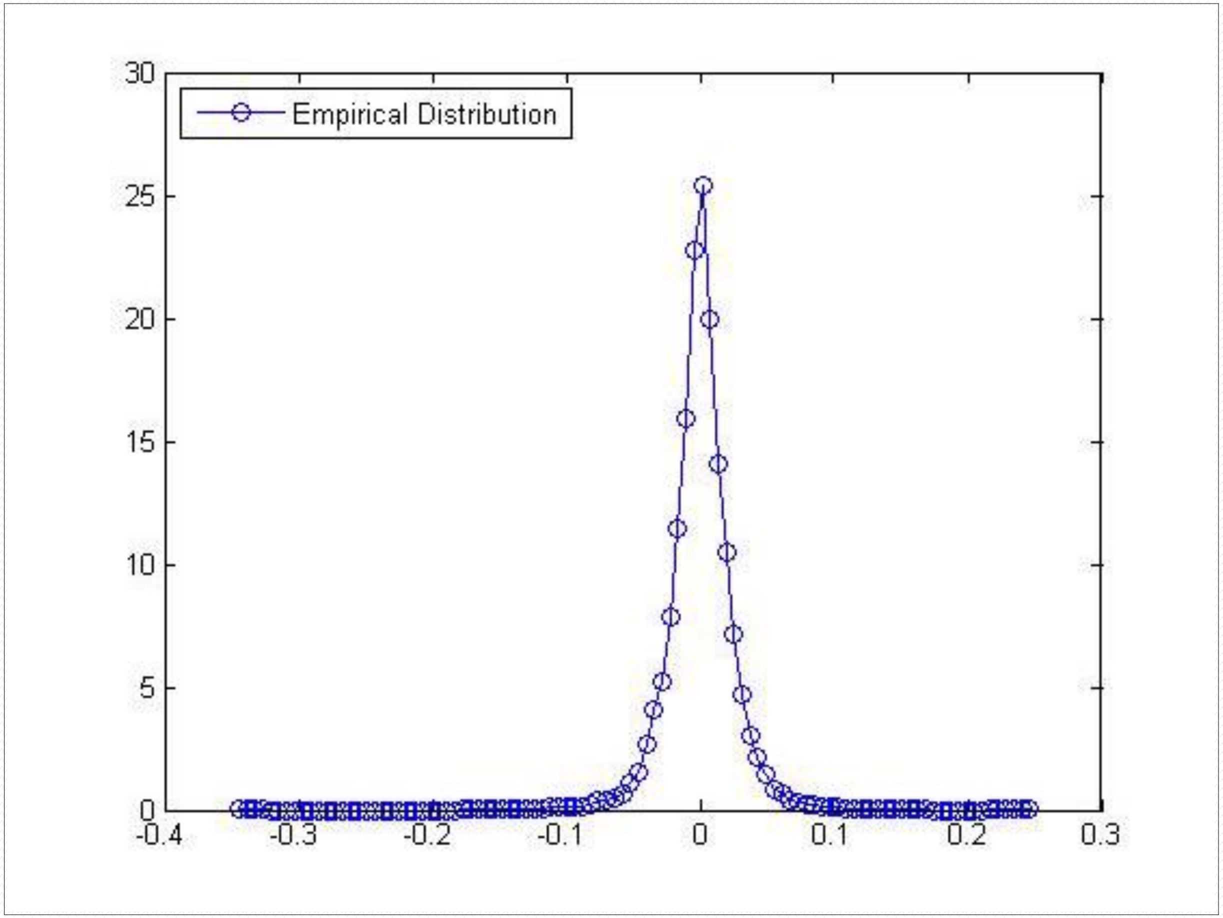


Figure 1: Empirical Distribution of Daily NYMEX Crude Oil (Front Month) Returns (1983-2010)

DIAGNOSTICS: RETURN ON NYMEX CRUDE OIL (FRONT MONTH)

Sample: 1979:01:05 - 2011:01:31

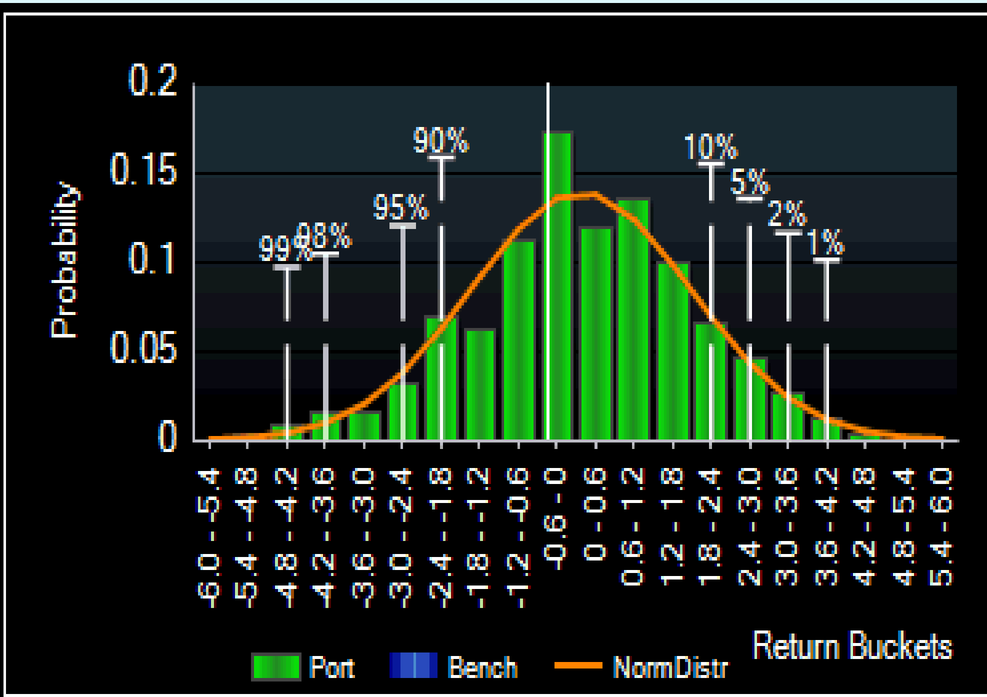
Mean	Stand Error	$Q(4)$	$Q^2(4)$	$Q(12)$	$Q^2(12)$
0.0004	0.025	13.9 [.007]	725 [.000]	72.7 [.000]	1410 [.000]

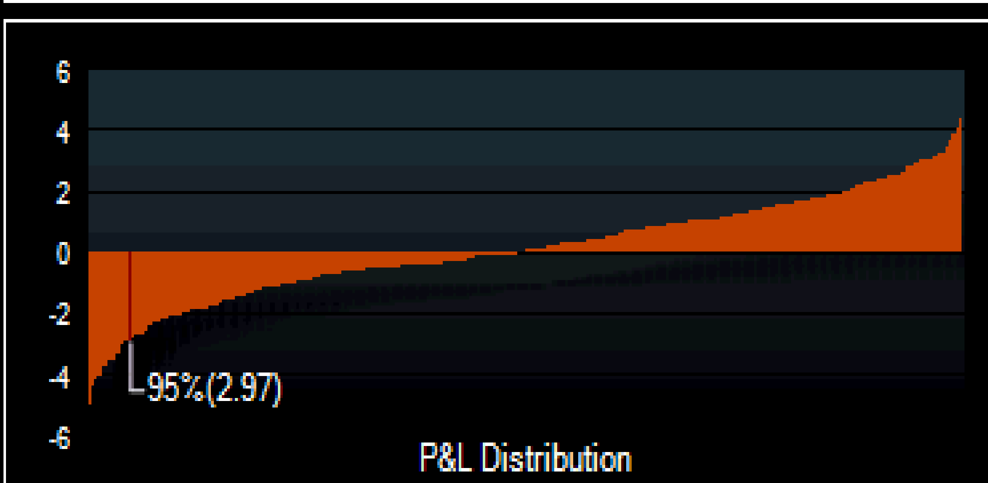
Skewness (Lev)	Kurtosis (Lev)	Skewness (Squ)	Kurtosis (Squ)
-0.219 [.000]	10.68 [.000]	23.35 [.000]	976 [.000]

Risk Measures On $\$USD/1000$ **barrels of Oil**

Historical		**Monte Carlo**		**Parametric (GARCH)**	
VaR 95%	CVaR 95%	VaR 95%	ETL 95%	VaR 95%	CVaR 95%
$4,267 (4.7%)	$6,541 (7.1%)	$4,710 (5.1%)	$6,687 (7.3%)	$2,013 (2.2%)	$2,194 (2.6%)

1 Year Best/Worst





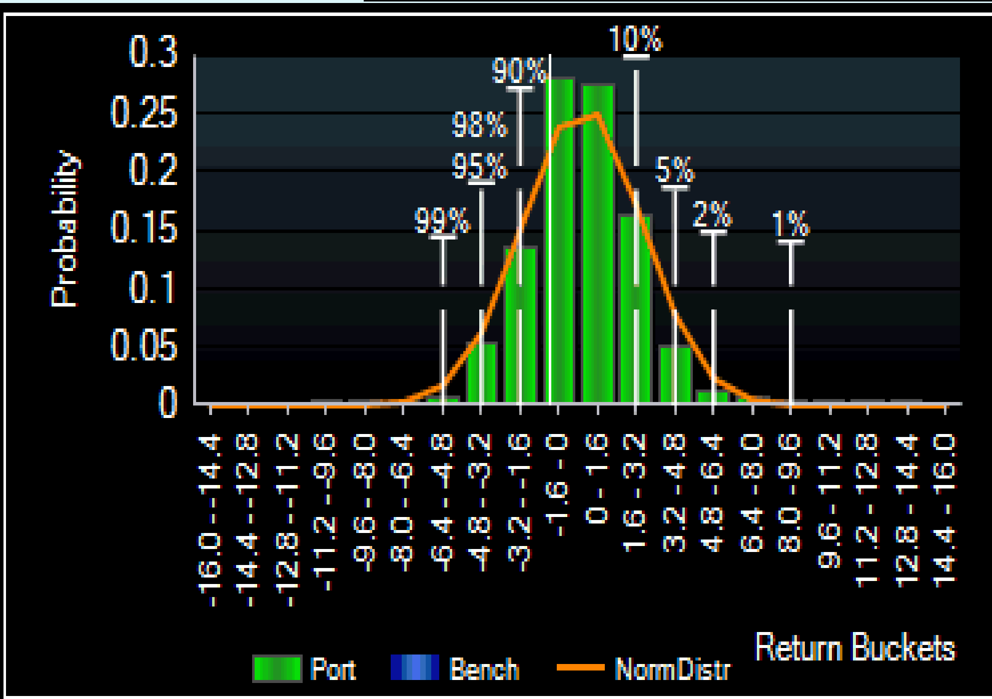
Best Days

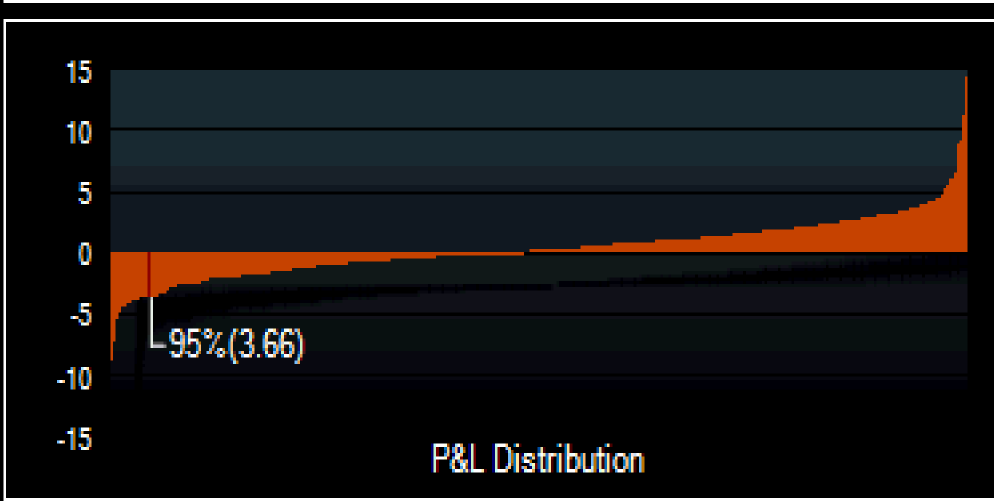
Date	P&L	Return
5/27/2010	3,919	4.32
5/26/2010	3,701	4.08
2/02/2010	3,468	3.82
7/22/2010	3,299	3.63
6/09/2010	3,061	3.37
11/24/2010	2,961	3.26
12/01/2010	2,894	3.19
6/25/2010	2,832	3.12
8/02/2010	2,791	3.07
5/21/2010	2,752	3.03

Worst Days

Date	P&L	Return
2/04/2010	-4,599	-5.07
10/19/2010	-3,984	-4.39
6/04/2010	-3,830	-4.22
5/04/2010	-3,690	-4.07
5/14/2010	-3,457	-3.81
8/31/2010	-3,431	-3.78
5/06/2010	-3,297	-3.63
7/01/2010	-3,267	-3.60
5/05/2010	-3,086	-3.40
11/12/2010	-3,076	-3.39

2 Years Best/Worst





Best Days

Date	P&L	Return
2/19/2009	12,942	14.26
3/12/2009	10,236	11.28
2/13/2009	9,577	10.55
3/04/2009	8,256	9.10
4/02/2009	8,097	8.92
3/19/2009	6,645	7.32
2/26/2009	5,900	6.50
2/25/2009	5,860	6.46
9/30/2009	5,390	5.94
4/09/2009	5,339	5.88

Worst Days

Date	P&L	Return
3/02/2009	-9,495	-10.46
4/20/2009	-8,151	-8.98
3/30/2009	-6,987	-7.70
3/11/2009	-6,817	-7.51
7/29/2009	-5,320	-5.86
2/12/2009	-5,028	-5.54
2/10/2009	-4,684	-5.16
2/04/2010	-4,599	-5.07
9/24/2009	-4,117	-4.54
7/08/2009	-4,087	-4.50

Tail Events & Crude Options

- **Example**
  - Market Price for June 2011 Future Contract: $97/barrel
  - A 5% OTM June 2011 Call Option expiring in 114 days is ~ $4.18
  - A 5% OTM June 2011 Put Option expiring in 114 days is ~ $4.18
  - **AND**
  - Implied vols for put is 29.877, and for call 28.5
  - Option seller collects premium of around 4.3% in each case
    - → Seller is wrong or "Normal" is ok?

Generalized Hyperbolic Distributions

Probability Density Function:

$$gh(l;\lambda,\alpha,\beta,\delta,\mu) = \alpha(\lambda,\alpha,\beta,\delta)[\delta^2 + (l-\mu)^2]^{(\lambda-\frac{1}{2})/2}$$
$$\times K_{\lambda-\frac{1}{2}}(\alpha[\sqrt{\delta^2 + (l-\mu)^2}]) \exp[\beta(l-\mu)]$$

where: $\alpha(\lambda,\alpha,\beta,\delta) = \frac{(\alpha^2-\beta^2)^{\lambda/2}}{\sqrt{2\pi}\,\alpha^{\lambda-\frac{1}{2}}\delta^\lambda K_\lambda(\delta\sqrt{\alpha^2-\beta^2})}$ And

$$\delta \geq 0,\ |\beta| < \alpha \text{ if } \lambda > 0$$
$$\delta > 0,\ |\beta| < \alpha \text{ if } \lambda = 0$$
$$\delta > 0,\ |\beta| \leq \alpha \text{ if } \lambda < 0$$

The parameters μ and δ are the location and the scale parameters, respectively.

Normal Inverse Gaussian Distribution

It is a subclass of the GHDs (for λ=-1/2) introduced by Eberlein and Keller (1995), and Barndorff-Nielsen (1995), with the density of

$$NIG(l;\alpha,\beta,\delta_t,\mu_t) = \frac{\alpha,\beta,\delta_t}{\pi}\exp[\delta_t\sqrt{\alpha^2-\beta^2}+\beta(l_t-\mu_t)]\frac{K_1(\alpha\sqrt{\delta_t^2+(l_t-\mu_t)^2})}{\sqrt{\delta_t^2+(l_t-\mu_t)^2}}$$

The NIG process $\boldsymbol{L_t}$ is a Levy process where its increments are distributed according to the distribution above.

Calibration and Simulation

Let St be the daily front month NYMEX crude oil prices at time t (*Eberlein and Raible (1999)*):

$$S_t = S_0 \exp(rt + L_t - wt)$$

where *r* is the interest rate, and

$$w = \mu + \delta\gamma - \delta\sqrt{\alpha^2 - (1+\beta)^2}$$

is a compensator term, to ensure that $S_t e^{-rt}$ is a martingale.

Calibration and Simulation

By Setting λ=-1/2 the calibration* results in

$$\mu = 0.00013, \alpha = 0.24201, \beta = 0.10001.$$

To Simulation the process, 100000 price path are generated (given market volatility of 30.81%)

***:** Multi-Cycle Expectation Conditional Maximization (See Meng and Rubin (1993), and Sexton and Swensen (2000)).

Calibrated GHDs vs Normal Distribution

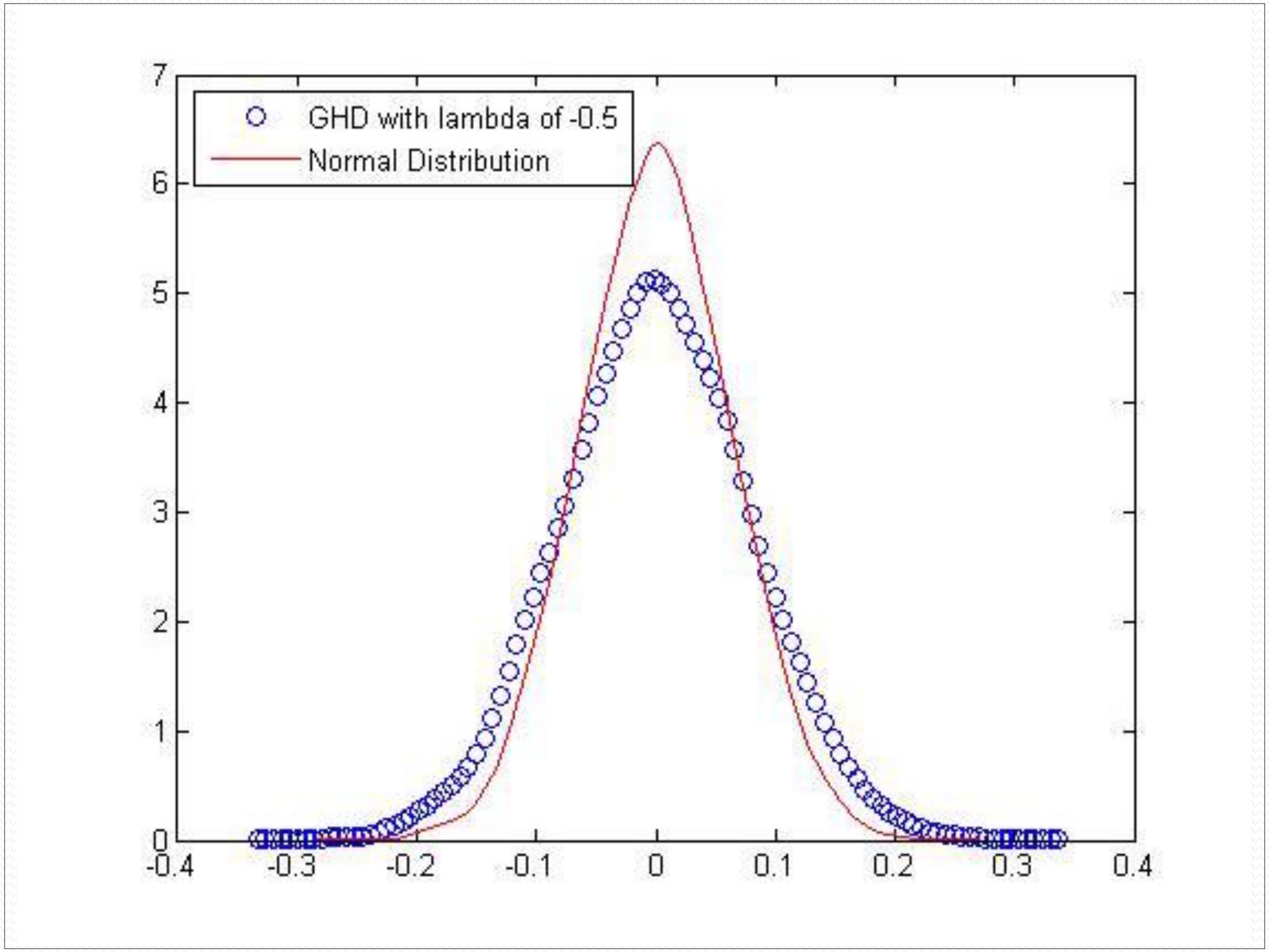


Figure 2: Calibrated GHD vs. Normal Distribution.

Simulation of St: Normal Dist and GHDs

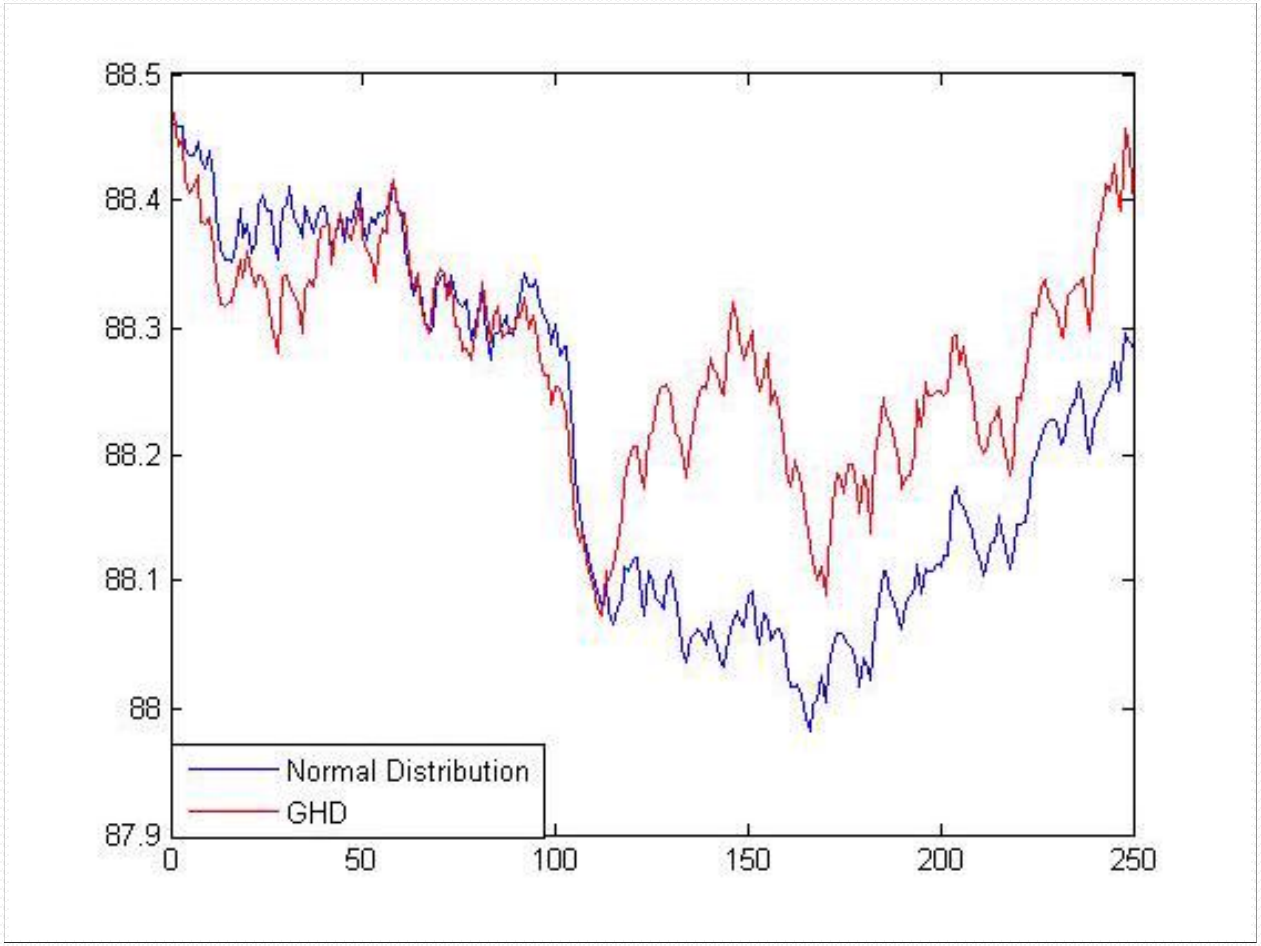


Figure 3: Simulation of (ref: st) under Normal and GHD (with lambda -0.5) distributions

Option Price Calculations

In a martingale frameword, the value of a European Call Option at time t is given by

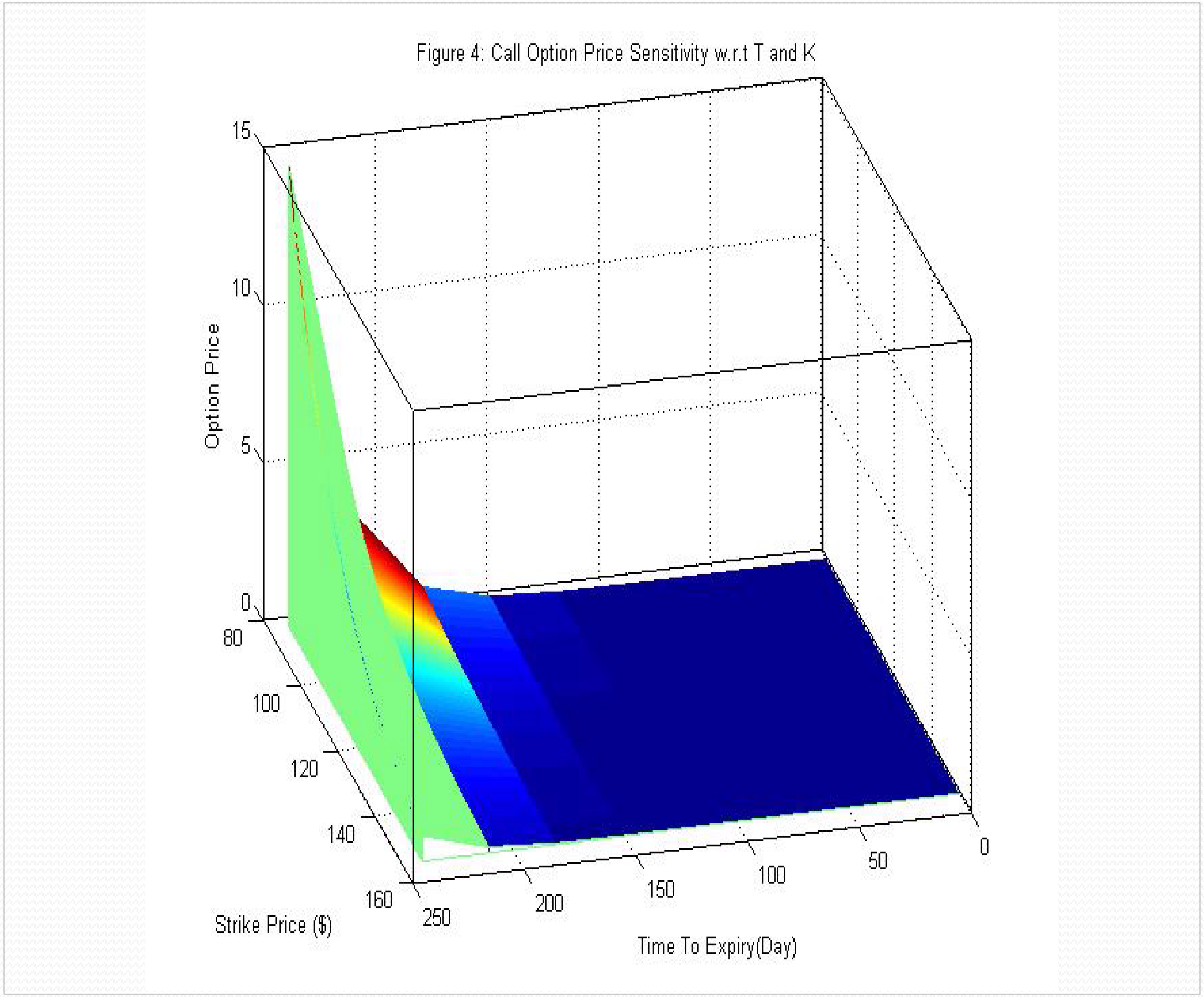
$$c_t = E_t[H_T e^{-r(T-t)}]$$

The Monte Carlo estimate of the value of option is approximated as

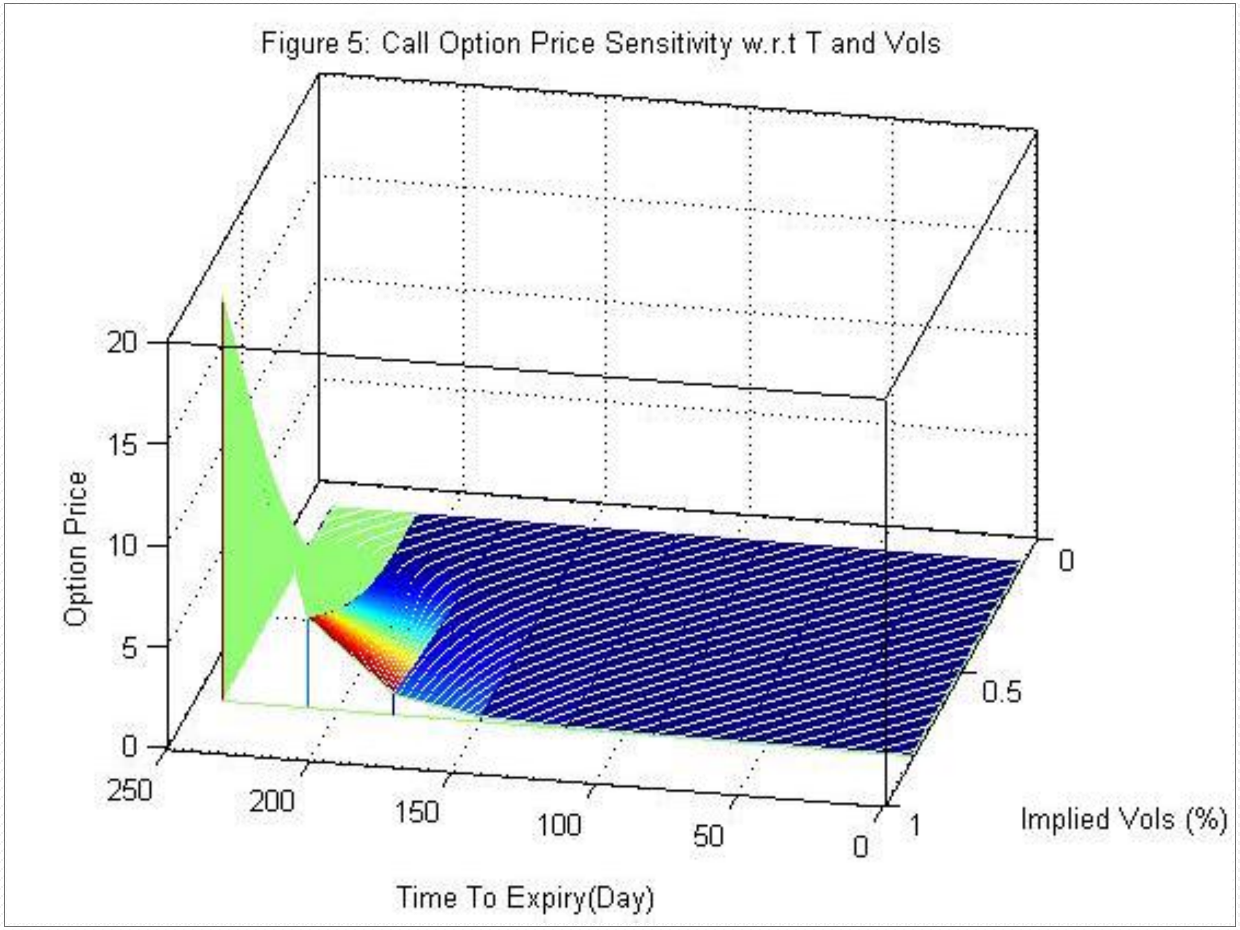
$$\hat{c}_t = e^{-r(T-t)} \frac{1}{T} \sum_{m=1}^{M} H_T(\hat{x}^m)$$

In constructing price path, we use $w=\mu+\delta\gamma-\delta\sqrt{(\alpha^2-(1+\beta)^2)}$ as a compensator term. Given implied volatility of 30.81%, interest rate of 5%, and underlying and strike prices of $91.2 the value of an at-the-money call option with 250 days to expiry is $11.011, which is almost $0.3 higher than Black-Scholes estimate, as expected.

Sensitivities w.r.t T & K



Sensitivities w.r.t T & Vols



Research Recommendations?

- What would be the Vol model of Option sellers vs Risk model of risk management system?
- Risk does not come from price itself! Alternative approach is to start from refinery margins, spreads, and inventories
- How does vols transition mechanisim work across future curve?