

VaR and CVaR: A Non-normal Regime Switching Framework

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Outline

① *Introduction.*

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- 2 *Methodology.*

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- 4 *Main Results.*

Purpose

- Develop a method for computing VaR and CVaR in a regime switching framework.

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- Capture the fat-tailed phenomenon by the Student's T distributions.
- Capture the volatility clustering phenomenon by the "regimes".

Definitions (1)

Definition

Risk: Roughly speaking, risk means a chance of injury or loss associated with a given action.

Definitions (2)

Definition

VaR is a statistical estimation of a portfolio loss with the property that, with a small probability α , the owner of the portfolio stands to incur that loss or more over a given (typically short) holding period.

Definitions (3)

Definition

CVaR equals the conditional expectation of X , given that $X > VaR_\alpha(X)$, i.e.

$$CVaR_\alpha = E[X | X > VaR_\alpha(X)].$$

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- Exchange Rates (Engle and Hamilton 1990)
- Short-term interest rates (Cai 1994, Gray 1996)

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- Guidolin and Timmermann (2003) provide a four state multivariate regime switching model for the joint distribution of monthly stock and bond returns. They found that the regime switching model is accurate in out-of-sample VaR predictions.

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- Kawata and Kijima (2005) introduce adjustment factors for state probabilities in regime switching models for estimate portfolio VaR, and argue that their models perform well.

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- We suppose the returns Y_t ($0 \leq t \leq T$) of an individual financial asset, S , follow a switching t-distribution, that is

$$Y_k \sim \begin{cases} T(\nu_1, \mu_1, \theta_1) & \text{at state 1,} \\ T(\nu_2, \mu_2, \theta_2) & \text{at state 2.} \end{cases}$$

Why Student's T Distribution

- Lin and Shen found that using the Student's T distribution for estimating VaR can improve the VaR estimation and offer accurate VaR estimates in their The Journal of Risk Finance (2006) paper.

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- We then change measure back to the real probability space.

EM Algorithm

The basic idea of the EM algorithm is:

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- 4 After more observations we repeat the process again.

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Thanks a lot for your time and attentions.

Suggestions and comments are appreciated.