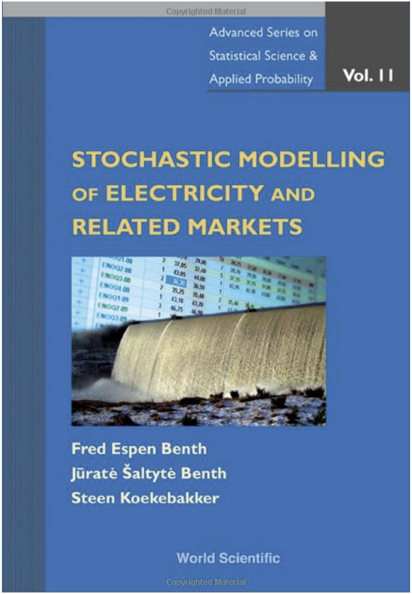

Chapter 7: Constructing smooth forward curves in electricity markets

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Introduction

The goal of this chapter is to represent forward prices by one continuous 'term structure' curve.

→ Useful for pricing (and marking to market) contracts whose settlement periods do not coincide with any traded contracts.

→ Necessary if no-arbitrage term structure models are to be used for risk management or (alternative) derivative pricing.

Fixed-income markets

- Literature spans 40 years, from McCulloch (1971) onwards.
- Main approaches involve fitting either a parametric function or a spline to observed yields.

Introduction

Electricity markets

- Data do not correspond to fixed-point yields, but to extended settlement periods.
- Because of non-storability, cost-of-carry relationships no longer hold and seasonality is a prominant feature.
- If the settlement period is long, the market data may not reflect the underlying seasonality, and the choice of seasonal function is a significant component of the modelling process.

Swap and forward prices

Basic approach

Their forward curves are constructed using

- a seasonal function (possibly formed using the MPS model of Botnen et. al. (1992)).
- an adjustment term, which they interpret as a market price of risk, and to which they apply smoothing techniques in line with existing work from fixed income markets (Adams and van Deventer (1994)).

Swap and forward prices

Relationships between swaps and forwards

$$F(t,\tau_1,\tau_2) = \int_{\tau_1}^{\tau_2} w(u,\tau_1,\tau_2) f(t,u)du,$$

where $w(u,s,t) = \dfrac{\hat{w}(u)}{\int_s^t \hat{w}(v)dv}$, with $\hat{w}(u) = 1$ or $\hat{w}(u) = \mathrm{e}^{-ru}$.
This holds in the idealized limit of continuous settlement. If settlement is discrete, at times u_i with intervals of Δ_i, then

$$F(t,\tau_1,\tau_2) = \sum_i w(u_i,\tau_1,\tau_2) f(t,u_i)\Delta_i.$$

The aim again is to extract $f(0,u)$ from observations of $F(0,\tau_1,\tau_2)$ for various values of τ_1 and τ_2.

Swap and forward prices

Creating a continuous curve

The basic model takes the form

$$f(u) = \Lambda(u) + \epsilon(u).$$

Λ is the seasonal function; ϵ captures the deviation from seasonality.

The authors assume that

- the seasonal function represents expectations under the objective measure.
- that the adjustment therefore captures market price of risk, as a function of time to delivery.
- that this function will become flat for the longest-dated contracts, so that $\epsilon'(u) \to 0$ as $u \to \infty$. They impose this via the constraint

$$\epsilon'(\tau_e) = 0.$$

Swap and forward prices

Creating a continuous curve

Additive case

To see how their model might relate to spot price models, they write, in the simple additive case,

$$S(t) = \Lambda(t) + X(t),$$

and then

$$f(u) = \Lambda(u) + \mathbb{E}_{\mathbb{Q}}[X(u)].$$

If X follows an OU process with constant coefficients, then the expectation under a risk-neutral probability parameterized by θ is

$$\mathbb{E}_{\theta}[X(u)] = \frac{\mu + \sigma\theta}{\alpha}(1 - \mathrm{e}^{-\alpha u}) + X(0)\mathrm{e}^{-\alpha u}.$$

This converges to a constant $\frac{\mu+\sigma\theta}{\alpha}$ as $u \to \infty$.
The additive form is simpler because it allows for consistent functional forms when integrating over settlement periods.

Maximum smoothness

- The authors construct ϵ from the space $C_0^2([\tau_\mathrm{s}, \tau_\mathrm{e}])$ of real-valued functions on $[\tau_\mathrm{s}, \tau_\mathrm{e}]$ which are twice continuously-differentiable and have zero derivative at τ_e.
- They choose ϵ to minimize

$$\int_{\tau_\mathrm{s}}^{\tau_\mathrm{e}} [\epsilon''(u)]^2 du$$

  from some subclass $\mathcal{C}$.
- In this work, $\mathcal{C}$ will consist of polynomial splines of order 4.
- They have to choose knots for the splines, and they do so using the set of all start and end times of traded contracts (in order, with no duplicates) $\tau_\mathrm{s} = \tau_0 < \tau_1 < \cdots < \tau_n = \tau_\mathrm{e}$.
- Thus

$$\epsilon(u) = a_i u^4 + b_i u^3 + c_i u^2 + d_i u + e_i, \quad u \in [\tau_{i-1}, \tau_i].$$

Constructing a smooth forward curve from closing prices

Set $\boldsymbol{x}' = \begin{bmatrix} a_1 & b_1 & c_1 & d_1 & e_1 & a_2 & b_2 & c_2 & d_2 & e_2 & \dots & a_n & b_n & c_n & d_n & e_n \end{bmatrix}$ and solve

$$\min_{\boldsymbol{x}} \int_{\tau_0}^{\tau_n} [\epsilon''(u;\boldsymbol{x})]^2 du$$

subject to

- continuity of $\epsilon(u;\boldsymbol{x})$, $\epsilon'(u;\boldsymbol{x})$ and $\epsilon''(u;\boldsymbol{x})$ at $\tau_1, \dots, \tau_{n-1}$,
- $\epsilon'(\tau_n;\boldsymbol{x}) = 0$ and
- $F_i^C = \int_{\tau_i^{\mathrm{b}}}^{\tau_i^{\mathrm{e}}} w(u, \tau_i^{\mathrm{b}}, \tau_i^{\mathrm{e}})(\epsilon(u;\boldsymbol{x}) + \Lambda(u))du, \quad i = 1, \dots, m$

  (i.e. for each traded contract with closing price F_i^C).

This constitutes a total of $3n + m - 2$ constraints, which can be formulated in the form $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$.

Constructing a smooth forward curve from closing prices

The minimization can be written as

$$\min_{\boldsymbol{x}} \boldsymbol{x}'\boldsymbol{H}\boldsymbol{x},$$

where (writing $\Delta_j = \tau_j - \tau_{j-1}$)

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{h}_1 & & 0 \\ & \ddots & \\ 0 & & \boldsymbol{h}_n \end{bmatrix}, \qquad \boldsymbol{h}j = \begin{bmatrix} \frac{144}{5}\Delta_j^5 & 18\Delta_j^4 & 8\Delta_j^3 & 0 & 0 \\ 18\Delta_j^4 & 12\Delta_j^3 & 6\Delta_j^2 & 0 & 0 \\ 8\Delta_j^3 & 6\Delta_j^2 & 4\Delta_j & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Lagrange multipliers can be used to solve this constrained problem, leading to the linear equation

$$\begin{bmatrix} 2\boldsymbol{H} & \boldsymbol{A}' \\ \boldsymbol{A} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{b} \end{bmatrix}.$$

Constructing a smooth forward curve from bid and ask prices

This time we have the constraint

$$F_i^B \leq \int_{\tau_i^{\mathrm{b}}}^{\tau_i^{\mathrm{e}}} w(u, \tau_i^{\mathrm{b}}, \tau_i^{\mathrm{e}})(\epsilon(u; \boldsymbol{x}) + s(u))du \leq F_i^A.$$

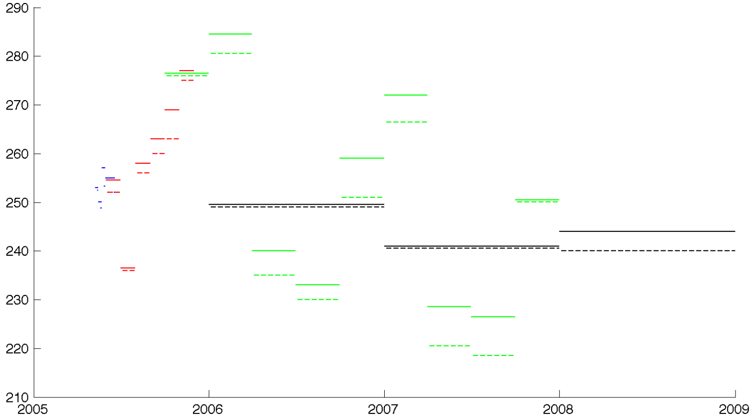
Non-binding constraints mean no Lagrange multipliers. In order to get around this, the authors use an iterative approach using pseudo closing prices moving at each iteration in a direction implied by the sign of the Lagrange multiplier.
The iteration stops if each F_i has hit the bid-ask boundary and the Lagrange multiplier would be pushing it outside the boundary.
Alternatively, a percentage improvement in the objective function smaller than some threshold can be used as a stopping criterion.

Nord Pool Example I

Table 7.1 Market data from Nord Pool, 4 May 2005

Ticker	Start date	End date	Bid	Ask
ENOW19-05	2005-05-09	2005-05-15	252.5	253.00
ENOW20-05	2005-05-16	2005-05-22	248.75	250.00
ENOW21-05	2005-05-23	2005-05-29	253.25	257.00
ENOW22-05	2005-05-30	2005-06-05	252.00	255.00
ENOW23-05	2005-06-06	2005-06-12	252.00	255.00
ENOW24-05	2005-06-13	2005-06-19	252.00	255.00
ENOMJUN-05	2005-06-01	2005-06-30	252.00	254.50
ENOMJUL-05	2005-07-01	2005-07-31	236.00	236.50
ENOMAUG-05	2005-08-01	2005-08-31	256.00	258.00
ENOMSEP-05	2005-09-01	2005-09-30	260.00	263.00
ENOMOCT-05	2005-10-01	2005-10-31	263.00	269.00
ENOMNOV-05	2005-11-01	2005-11-30	275.00	277.00
FWV2-05	2005-10-01	2005-12-31	276.00	276.50
ENOQ1-06	2006-01-01	2006-03-31	280.50	283.50
ENOQ2-06	2006-04-01	2006-06-30	235.00	240.00
ENOQ3-06	2006-07-01	2006-09-30	230.00	233.00
ENOQ4-06	2006-10-01	2006-12-31	251.00	259.00
ENOQ1-07	2007-01-01	2007-03-31	266.50	272.00
ENOQ2-07	2007-04-01	2007-06-30	220.50	228.50
ENOQ3-07	2007-07-01	2007-09-30	218.50	226.50
ENOQ4-07	2007-10-01	2007-12-31	250.00	250.50
ENOYR-06	2006-01-01	2006-12-31	249.00	249.50
ENOYR-07	2007-01-01	2007-12-31	240.50	241.00
ENOYR-08	2008-01-01	2008-12-31	240.00	244.00

Nord Pool Example I

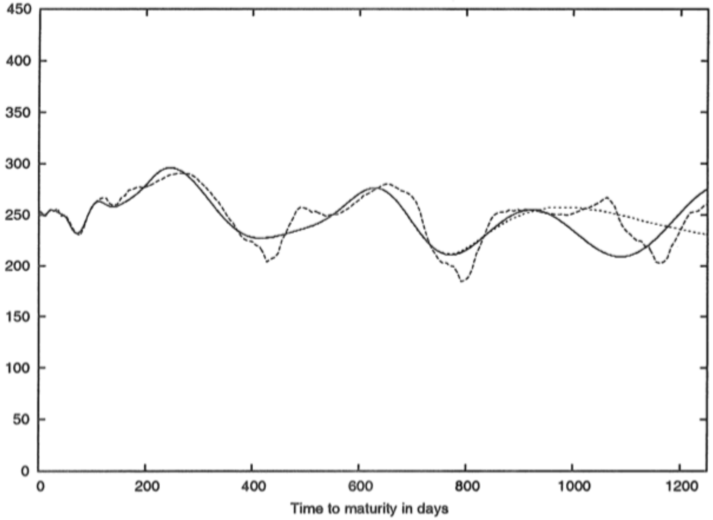


Nord Pool Example I

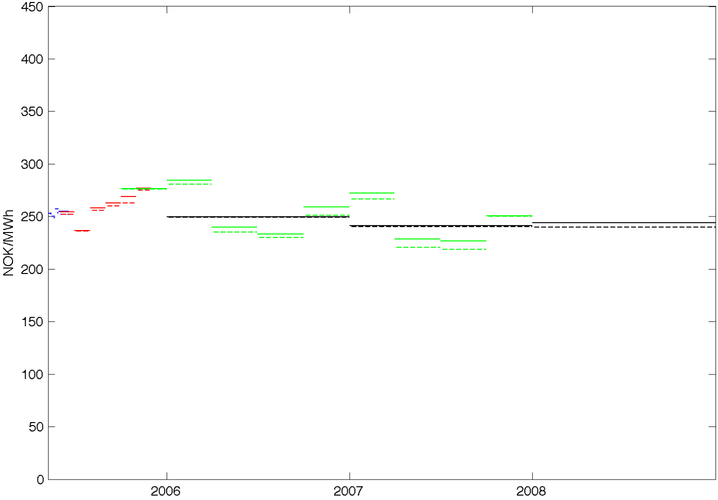
Seasonality specifications

- $\Lambda(u) = 0$.
- $\Lambda(u) = 145.732 + 29.735 \cos\left((u + 6.691)\frac{2\pi}{265}\right)$ (Lucia and Schwartz (2002)).
- spot prognosis from a bottom-up model (from Adger Energi).

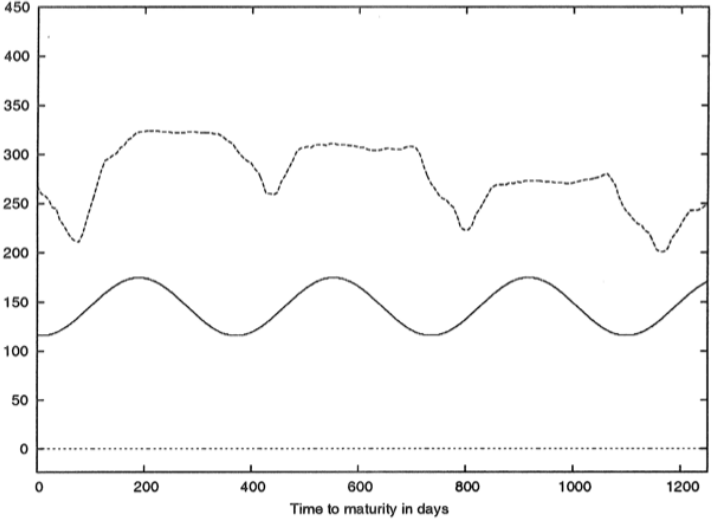
Nord Pool Example I: smoothed forward curves



Nord Pool Example I: smoothed forward curves



Nord Pool Example I: seasonality functions



Nord Pool Example II: analyzing volatility

Table 7.2 Maturities of forwards and delivery periods for electricity futures (in days)

m	u_j	τ_j^b	τ_j^e
1	10.5	7	14
2	17.5	14	21
3	24.5	21	28
4	31.5	28	35
5	38.5	35	42
6	45.5	42	49
7	52.5	49	56
8	72	56	86
9	101	86	116
10	131	116	146
11	161	146	176
12	191	176	206
13	221	206	236
14	251	236	266
15	281	266	296
16	311	296	326
17	341	326	356
18	401	356	446
19	491	446	536
20	581	536	626
21	671	626	716
22	896	716	1076

- For each of 1076 trading days onstruct two forward curves using zero seasonality and Lucia-Schwartz seasonality.
- Extract 22 forward and futures prices corresponding to Table 7.2.
- Model forward curve using $df(t,u) = \sigma(t,u)dW(t)$, or
- Model swap contracts using $dF(t,\tau_1,\tau_2) = \Sigma(t,\tau_1,\tau_2)dW(t)$, where $\Sigma(t,\tau_1,\tau_2) = \int_{\tau_1}^{\tau_2} w(u,\tau_1,\tau_2)\sigma(t,u)du$.
- There may be non-zero drift terms in the objective measure, but we can use observations (if they are continuous) to estimate σ and Σ.

Nord Pool Example II: analyzing volatility

Table 7.2 Maturities of forwards and delivery periods for electricity futures (in days)

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5	38.5	35	42
6	45.5	42	49
7	52.5	49	56
8	72	56	86
9	101	86	116
10	131	116	146
11	161	146	176
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13	221	206	236
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- Set

$$x_{i,j}^f = f(t_i, u_j) - f(t_{i-1}, u_j) \approx df(t_i, u_j)$$

and

$$x_{i,j}^F = F(t_i, \tau_j^b, \tau_j^e) - F(t_{i-1}, \tau_j^b, \tau_j^e) \approx dF(t_i, \tau_j^b, \tau_j^e).$$

- Compute

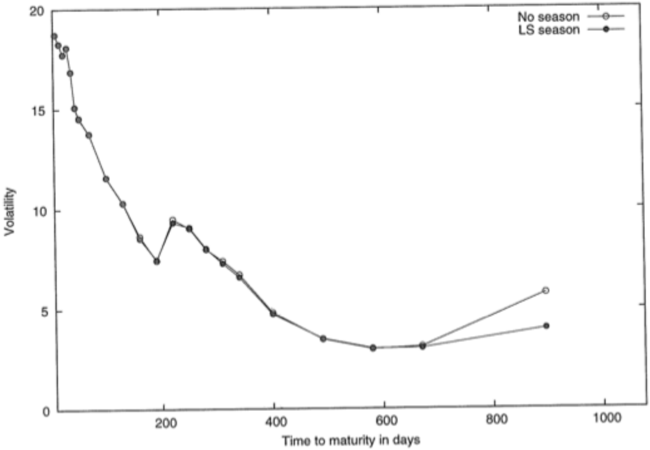
$$\widehat{\Sigma}_j = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_{i,j}^F - \bar{x}_j^F)^2}$$

and

$$\widehat{\sigma}_j = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_{i,j}^f - \bar{x}_j^f)^2}$$

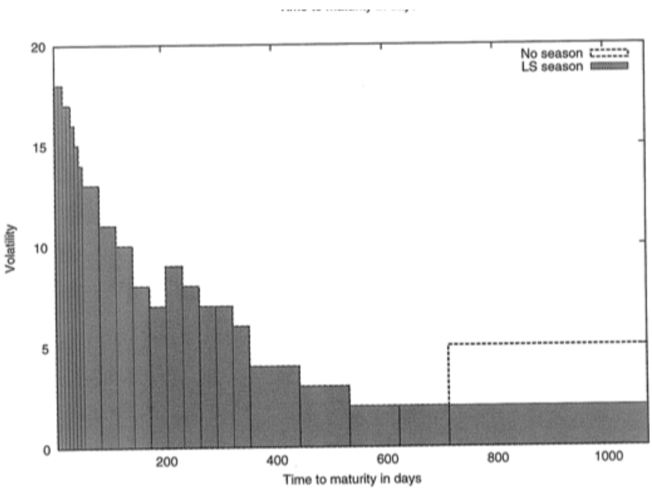
Nord Pool Example II: analyzing volatility

Volatility estimates for forward prices ($\widehat{\sigma}_j$)



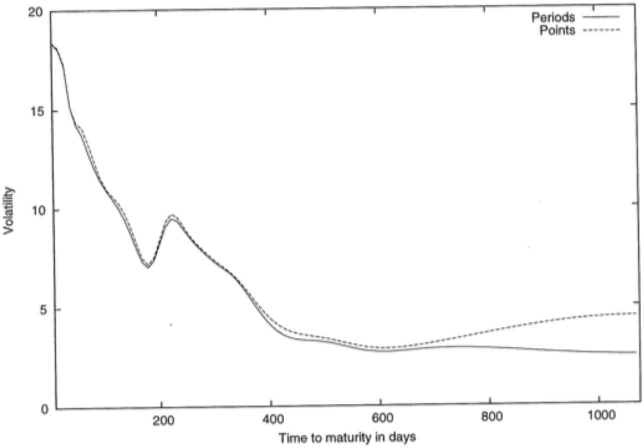
Nord Pool Example II: analyzing volatility

Volatility estimates for futures prices ($\widehat{\Sigma}_j$)



Nord Pool Example II: analyzing volatility

Smoothed forward volatility curves with trigonometric seasonality



Summary

- The specification of $\Lambda(u)$ affects the appearance of the forward curve when the market information involves swaps with long delivery periods.
- Ignoring seasonality leads to an upward-biased volatility estimate in the long end, although using swap prices rather than forward prices lessens this effect.
- An arithmetic model was used so that the problem was tractable and direct comparisons could be made between using swaps and using forwards.