Review Three Papers by F. Benth et. al

2005-2007

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Outline

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Paper #1 : A Non-Gaussian Ornstein-Uhlenbeck Process for Electricity Spot Price Modeling and Derivatives Pricing

Fred Espen Benth

Jan Kallsen

Thilo Meyer-Brandis

May 2005

Introduction

- Aim is to propose a model for the electricity spot dynamics
- A Non-Gaussian Ornstein-Uhlenbeck Process
  - Sum of O-U processes
  - Each reverting to mean at a different speed
  - Having pure jump processes with only positive jumps as sources of randomness
- Additive structure and gives positive prices

Electricity markets

Electricity exchanges organize trade in:

- Hourly spot electricity next-day delivery
- Financial forward/futures contracts
- European options on forwards

In particular, from a methematical finance point of view

- Spot electricity: non-storability of electricity renders markets highly incomplete underlying not tradable
- Forwards: Delivery of electricity over period of weeks/months/quaters of year rather then fixed delivery

Electricity price modelling

Two categories of approaches for electricity price modelling:

1. Direct modelling of futures prices
   - Transfer of concepts from interest rate theory(HJM approach) (Clewlow Strickland 1999, Manoliu Tompaidis 2002, Benth Koekebakker 2005)
   - Advantage: complete market and risk neutral pricing machinery available
   - Problem: no inference about spot prices possible (arbitrage relations not valid)

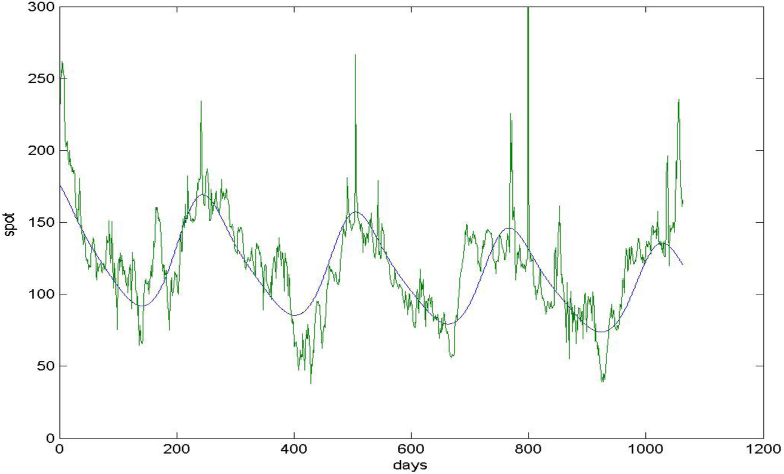
Electricity price modelling

2. Spot price modelling:
   - Various structured OTC products depend on spot evolution: spot price model required
   - Use spot price model to derive prices of futures (and other derivatives)
   - breakdown of spot-futures relationship: identification of market price of risk (pricing measure) necessary to derive futures prices

In this work we want to introduce a model of the second category, i.e. a new electricity spot price model.

Essential features of spot prices

Daily NordPool system price 01.1997 – 07.2000



Essential features of spot prices

Stylized features of electricity spot prices are:

- mean reversion
- seasonality
  - yearly price cycle (in example above winter has higher prices than summer)
  - weekly seasonality
  - intra-daily cycles
- intrinsic feature of sharp spikes followed by sharp drops (leptokurtic returns)
- level dependent volatility

Modelling requirements of spot dynamics

A spot price model should

- reflect statistics and path properties of historical data
- reflect physical conditions and constraints
- but also allow for sufficient analytical tractability:
  - risk evaluation
  - forward/futures price dynamics
  - option pricing

In particular, analytical pricing of forwards and futures is very desirable.

Common spot price models

Most common reduced form spot price models are of exponential Ornstein-Uhlenbeck type

- guarantees positive prices
- enhances robustness of calibration procedure

However

- Is the exponential structure the right transformation for electricity prices?
  - exponential structure originates from population growth modelling (in finance compound interest modelling)
- Most importantly, no manageable analytic expressions for corresponding forward/futures contracts!

An arithmetic model

We propose to model the spot price as a sum of non-Gaussian OU-processes:

$$S(t) = \mu(t) + \sum_{i=1}^{n} \omega_i Y_i(t)$$

where

$$dY_i(t) = -\lambda_i Y_i(t)dt + \sigma_i(t)dL_i(t)$$

- $L_i(t)$ are independent increasing time inhomogeneous pure jump Lévy processes (additive processes).
- We suppose a Lévy measure of $L_i(t)$ of the form

$$\nu_i(dt, dz) = \rho_i(t)dt\nu_i(dz)$$

where $\rho_i(t)$ controls seasonal variation of jump intensity.

An arithmetic model

- $\sigma_i(t)$ controls seasonal variation of jump sizes
- λ_i is different level of mean reversion
- $\mu(t)$ is deterministic seasonality function

$\rightarrow$ The model guarantees positive prices because the $L_i(t)'s$ are increasing.
$\rightarrow$ Upward jumps are followed by downward drops whose sharpness is controlled by the corresponding λ_i .
$\rightarrow$ The model allows for analytical pricing of corresponding forward and fututres contracts.

Pricing of forward/futures contracts

- Let $F(t; T_1, T_2)$ be time t forward price of a contract which delivers electricity at a rate $S(t)/T_2 - T_1$ during the settlement period $[T_1, T_2]$:

$$\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} S(u) du.$$

- Forward price defined so that time t value is zero, given information about the spot price up to time t:

$$F(t; T_1, T_2) = \mathbb{E}_Q \Big[\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} S(u) du | \mathcal{F}_t \Big],$$

where Q is a pricing measure to be determined.

Pricing of forward/futures contracts

Proposition:

$$F(t; T_1, T_2) = F(0; T_1, T_2) + \sum_{i=1}^{n} \frac{1}{\lambda_i(T_2 - T_1)} \int_0^t \sigma_i(s)(e^{-\lambda_i(T_1-s)} - e^{-\lambda_i(T_2-s)}) d\bar{L}_i(s)$$

where

$$F(0; T_1, T_2) = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} \left\{ \mu(u) + \sum_{i=1}^{n} \omega_i \left(y_i e^{-\lambda_i u} + \int_0^u \int_{\mathbb{R}^+} \sigma_i(s) e^{-\lambda_i(u-s)} z \hat{\nu}_i(dz, ds) \right) \right\} du$$

and $\bar{L}_i(t)$ is the compensated jump process with compensating measure $\hat{\nu}_i(dz, ds)$ under Q.

Pricing of options on forward/futures contracts

- Some notations:

$$\sum_i(t, T_1, T_2) = \frac{\sigma_i(t)}{\lambda_i(T_2 - T_1)}(e^{-\lambda_i(T_1-s)} - e^{-\lambda_i(T_2-s)}).$$

$$\begin{aligned}\tilde{\psi}^i_{t,T}(\theta) &= ln\mathbb{E}_Q[exp(i\int_t^T \theta(s)dL_i(s))] \\ &= \int_t^T \int_0 \left\{e^{i\theta(s)z} - 1\right\}\hat{\nu}_i(dz, ds)\end{aligned}$$

- Let $g \in L^1(R)$ be payoff of an option written on $F(T; T1, T2)$, $T \leq T1$. Then the price is given by

$$p(t; T; T_1, T_2) = e^{-r(T-t)}\mathbb{E}_Q[g(F(T; T_1, T_2))|F_t].$$

Pricing of options on forward/futures contracts

Proposition:
If $g(F(T, T1, T2)) \in L^1(Q)$, then we have that

$$p(t; T; T_1, T_2) = e^{-r(T-t)}(g \star \Phi_{t,T})(F(T; T_1, T_2))$$

where the function $\Phi_{t,T}$ is defined via its Fourier transform

$$\hat{\Phi}_{t,T}(y) = exp(\sum_{i=1}^{n} \tilde{\psi}^i_{t,T}(y \sum_i(\cdot, T_1, T_2))),$$

and $\star$ is the convolution product.

- Numerical pricing by fast Fourier transform techniques.
- Not available for exponential models in this explicit form.
- Exponential damping for payoffs not in $L^1(R)$ (e.g. Carr Madan 1999).

Conclusion

- Most common spot models are of geometric type and become unfeasible for further analysis of derivatives pricing.
- We propose an arithmetic model that is simple enough to yield analytical forward prices. Option pricing by fast Fourier transform techniques.
- The arithmetic model describes well both path properties and statistics of electricity spot prices.
- Future work includes the calibration of the market price of risk and the study of futures prices induced by the model.

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Paper #2 : Pricing of Exotic Energy Derivative Based on Arithmetic Spot Models

Fred Espen Benth

Rodwell Kufakunesu

September 2007

Introduction

- Energy markets are trading exotic derivatives in large volumes
- Important examples like spark and dark
  - Spark spread is the difference between spot electricity and gas
  - Dark spread refers to the difference between electricity and coal
- To get reliable prices of derivatives, one needs to have accessible spot price models which incorporate the stylized facts of energy prices
- It is desirable to have models which are analytically tractable for derivatives pricing
- Proposed model is arithmetic, defined to ensure the positivity of spot prices, a desirable feature in the market

Literature

- Margrabe(1978) gives an explicit expression for the price of a call option with zero strike on the spread between two financial assets following geometric motions.
- Carmona and Durrleman(2003) present a detailed study of pricing of spreads based on diffusion models, approxiamtion formula are provided.
  - excellent introduction to the zoology of spread options traded in the market.
- Dempster and Hong(2000) propose and analyse a fast Fourier transform method for spread options based on a two-dimensional geometric Brownian motion.
  - method involves ingenious approximations of a two-dimensional Fourier transform.
- Benth and Saltyte-Benth(2006) have proposed model the spart spread directly using an arithemtic jump-diffusion model.
- Benth and Kettler (2006) have used copula theory to model the spark spread, and Monte Carlo simulated the price of put and call options.

An Arithmetic Model for the Spot Price

Benth, Kallsen and Meyer-Brandis (2005) proposed to model the spot price as a sum of non-Gaussian OU-processes:

$$S(t) = \Lambda(t) + \sum_{i=1}^{n} Y_i(t)$$

where

$$dY_j(t) = -\lambda_j Y_j(t)dt + \sigma_j(t)dL_j(t)$$

- L_j is an inhomogeneous subordinator, i.e. an IIP with only positive jumps and no continuous martingale part.
- Λ is a deterministic seasonal floor function.
- λ_j and σ_j are positive continuous functions on $[0, T]$.
- λ_j are time-dependent for generality.

An Arithmetic Model for the Spot Price

An explicit representation of $Y_j(u)$ for $u \geq t \geq 0$ is

$$Y_j(u) = Y_j(t)e^{-\int_t^u \lambda_j(s)ds} + \int_t^u \sigma_j(s)e^{-\int_s^u \lambda_j(v)dv} dL_j(s).$$

- All the OU-processes will mean-revert towards to zero, implies that the spot price mean-reverts to the seasonal level function $\Lambda(t)$.
- Expected spot price will be a sum of the expectations of Y_j and the floor.
- Need to assume the floor function is positive.
- Let some of the $Y_j's$ account for the normal variations in the market, while one or two represent the spikes.

Asian Options

- Consider average-type options, or Asian options, written on an energy spot price $S(t)$
- Suppose the option pays $g(\int_\tau^T S(u)du)$, $\tau < T$, at maturity T, with $g \in L^1(R_+)$

Price $C(t)$ of the option at time t is defined from the arbitrage theory to be

$$C(t) = e^{-r(T-t)}\mathbb{E}_\theta\Big[g(\int_\tau^T S(u)du)|\mathcal{F}_t\Big].$$

Asian Options

Proposition:
The Price $C(t)$ is given as

$$C(t) = \frac{e^{-r(T-t)}}{2\pi} \int_R \hat{g}(y)\Psi(t,\tau,T,y,\theta)dy,$$

where Ψ is defined by

$$ln\Psi(t,\tau,T,y,\theta) = iy\int_\tau^T \Lambda(u)du + iy\sum_{j=1}^{n}\int_\tau^T e^{-\int_t^u \lambda_j(v)dv}duY_j(t$$

$$+\sum_{j=1}^{n}\psi_j(t,T;y\sigma_j(\cdot)\int_{max(\cdot,\tau)}^{T} e^{-\int_\cdot^u \lambda_j(v)dv}du - i\theta_j(\cdot)) - \psi_j(t,T;-i\theta_j(\cdot))$$

Asian Options Example (Put Options)

Typical examples of Asian options are plain vanilla put and call options written on the average of the spot.
Consider a put option:

- Payoff function $g(x) = max(K - x, 0)$ for $x \geq 0$
- Extending g with $g(x) = 0$ for $x < 0$, we can invoke the result in Proposition.
- Calculation shows that the Fourier transform of g in the put option case is

$$g(\hat{y}) = \frac{iyK - e^{-iyK} + 1}{y^2}.$$

- An FFT algorithm is then can be used to calculate $C(t)$ efficiently on a computer
  - We need to know the cumulant functions ψ_j
  - An appropriate numerical integration must be invokeed in the calculations of the terms.

Asian Options Example (Call Options)

- Payoff function $g(x) = max(x - K, 0)$ for $x \geq 0$
- Extending g with $g(x) = 0$ for $x < 0$.
- Not integrable on R due to the linear growth. $\Rightarrow$ cannot use Proposition.

Alternatives:

- Dampen the payoff function with an exponential function in order to obtain something integrable. (Carr and Madan, "Option valuation using the fast Fourier transform", 1998).
- Appeal to the put-call parity.

Spread Options

- Spread usually involve two energies.
- Extending the spot model to a bivariate case, include both unique and common risk.
- Some properties will be studied first before move on to the price of spread options
- Include the case of baskets.

Spread Options

Consider two energies A and B, with spot price dynamics defined by

$$S^A(t) = \Lambda^A(t) + \sum_{i=1}^{m} X_i^A(t) + \sum_{j=1}^{n} Y_j^A(t)$$

$$S^B(t) = \Lambda^B(t) + \sum_{i=1}^{m} X_i^B(t) + \sum_{j=1}^{n} Y_j^B(t)$$

We suppose that the first m factors X_i^A and X_i^B are common, in the sense that the OU-processes are driven by the same jump processes.
Hence, we suppose that:

$$dX_i^A(t) = -\alpha_i^A X_i^A(t)dt + \sigma_i^A dL_i(t)$$

$$dX_i^B(t) = -\alpha_i^B X_i^B(t)dt + \sigma_i^B dL_i(t)$$

where α_i^A, α_i^B, σ_i^A and σ_i^B are positive constants, and the IIP $L_i, i = 1, 2, ...m$ are independent.

Spread Options

For simplicity, we have dispensed with the general time-dependent mean-reversion and variation coefficients $\alpha' s$ and $\sigma' s$ and consider only constants. Further, we let

$$dY_j^A(t) = -\beta_j^A Y_j^A(t)dt + \eta_j^A dL_j^A(t)$$

$$dY_j^B(t) = -\beta_j^B Y_j^B(t)dt + \eta_j^B dL_j^B(t)$$

where all parameters are positive constants, and L_j^A and L_j^B are IIP being mutually independent.

Spread Options

Condition(I): There exists a constant $k_j > 0$ such that the compensator measure $l_j(dz, ds)$ satisfies the integrability condition:

$$\int_0^T \int_1^\infty e^{zk_i} l_j(dz, ds) < \infty$$

Proposition:
Suppose that Condition (I) holds for constant k bigger than or equal to 2 for all the compensator measures. Then the covariance between $S^A(t)$ and $S^B(t)$ is

$$Cov(S^A(t), S^A(t)) = \sum_{i=1}^{m} \sigma_i^A \sigma_i^B \int_0^t \int_R z^2 e^{-\alpha_i^A + \alpha_i^B}(t-s) l_i(dz, ds).$$

Spread Options

If the common factors $L_i, i = 1, 2, ...m$ are subordinators, we have:

$$Cov(S^A(t), S^A(t)) = \sum_{i=1}^{m} \frac{\sigma_i^A \sigma_i^B}{\alpha_i^A + \alpha_i^B} \left(1 - e^{-\alpha_i^A + \alpha_i^B t}\right) \int_R z^2 l_i(dz),$$

since the compensator measures then are $l_i(dz, ds) = l_i(dz)ds$.
Letting $t \to \infty$, we derive a stationary covariance function:

$$Cov(S^A(t), S^A(t)) \approx \sum_{i=1}^{m} \frac{\sigma_i^A \sigma_i^B}{\alpha_i^A + \alpha_i^B} \int_R z^2 l_i(dz).$$

European Options

- Exercise time T and payoff function $g(S(T))$.
- Contrary to the Asian option, we can have that $S(T)$ is negative.
- The price $C(t)$ at time t of the option is then defined by

$$C(t) = e^{-r(T-t)}\mathbb{E}_\theta[g(S(t))|\mathcal{F}_t].$$

- The price may be expressed in terms of the cumulant functions of the jump processes.

European Options

Proposition:
The price $C(t)$ of a European option with payoff $g(S(t))$ at exercise time $T > t$ is

$$C(t) = \frac{e^{-r(T-t)}}{2\pi} \int_R \hat{g}(y)\Psi(t, T, y, \theta)dy$$

European Options

where

$$
\begin{aligned}
ln\Psi(t, T, y, \theta) = \; & iy(a\Lambda^A(T) + b\Lambda^B(T)) \\
& + iy \sum_{i=1}^{m} aX_i^A(t)e^{-\alpha_i^A(T-t)} + bX_i^B(t)e^{-\alpha_i^B(T-t)} \\
& + iy \sum_{j=1}^{n} aY_j^A(t)e^{-\beta_j^B(T-t)} + bY_j^B(t)e^{-\beta_i^B(T-t)} \\
& + \sum_{i=1}^{m} \Big(\psi_i(t, T; y(a\sigma_i^A e^{-\alpha_i^A(T-\cdot)} + b\sigma_i^B e^{-\alpha_i^B(T-\cdot)} - i\theta(\cdot))) \\
& \qquad - \psi_i(t, T; -i\theta_i(\cdot)) \Big) \\
& + \sum_{j=1}^{n} \psi_j^A(t, T; y(a\eta_j^A e^{-\beta_j^A(T-\cdot)} - i\theta_j^A(\cdot))) - \psi_j^A(t, T; -i\theta_j^A(\cdot)) \\
& + \sum_{j=1}^{n} \psi_j^B(t, T; y(a\eta_j^B e^{-\beta_j^B(T-\cdot)} - i\theta_j^B(\cdot))) - \psi_j^B(t, T; -i\theta_j^B(\cdot)
\end{aligned}
$$

European Options

- The expression is suitable for the FFT metod, as long as we know the cumulant functions and Fourier transform of g.
- Calculation only involves a one-dimensional inversion of the Fourier transform.

Compare to geometric models

- i.e. the mean reverting model proposed by Schwartz
- Need calculate a two-dimensional inverse Fourier transform.
- The expression become considerably more intricate, as shown by Dempster and Hong (2000)

Conclusions and Open Problems

- Asian and spread options can be easily calculated by FFT for the positive arithemtic spot price model.
- Expressions are derived based on the characteristic functions of the driving stochastic innovations, being IIP.
- A key to obtaining market relevant option prices is the determination of the market price of risk.
- Options we have consiered are mostly traded OTC, making it hard to obtain good and reliable data, this complicated our approach.
- We have supposed a simple deterministic form of the market price of risk, but it may be stochastic, making the problem of pricing even harder.

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Paper #3 : Pricing Forward Contracts in Power Markets by the Certainty Equivalence Principle: explaining the sign of the market risk premium

Fred Espen Benth

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December 2007

Introduction

- Commodities are a very different asset class from equities and bonds.
- Physical nature plays an important role in the behavior of their prices in both the spot and forward market.
- Contrast equity forwards with commodity forwards:
  - Pricing equity forwards is straightforward if interest rates and dividends are deterministic.
  - In principle, similar strategy can be apply to commodity, however, very difficult:
    - ★ the cost-of-carry is not straightforward to calculate or measure.
    - ★ the convenience yield is exceptionally difficult to quantify or model.

Commodities' forward curves

- It is the producers, consumers and speculators attitudes' towards bearing risk in the markets
- Forwards exhibit peculiar behavior depending on the time or delivery period.
- Contango: forward prices are above current spot prices
  - immediate supply of the commodity is plentiful relative to demand.
- Backwardation: forward prices are below spot prices
  - low current supply levels and / or low inventory levels.
- For long dated forward contracts, markets are in backwardation.
- For shorter maturities the market is in contango.
  - See Álvaro Cartea and Marcelo G. Figueroa : Pricing in electricity markets, 2005 for detail.

Market Risk Premium

- Defined as the difference, calculated at time t, between the forward price $F(t, T)$, at time t with delivery at T, and the expected spot price:

$$\pi(t, T) = F(t, T) - \mathbb{E}^P[S(T)|\mathcal{F}_t].$$

- $\mathbb{E}^P$ is the expecation operator, under the historical measure P, with information up until time t.
- $S(t)$ is the spot price at time T.

Market Risk Premium

- In literature, no connection between the market risk premium and market players' behavior and risk preferences has been addresses.
- Why and how the market risk premium $\pi(t, T)$ changes sign in time T in some commodities market.
- Main contribution of the paper:
  - Address the above question
  - Propse a framework that allows us to establish explicit relationships between the market risk premium, the market price of risk and market players' risk preferences.

Market Risk Premium

- Positive market risk premium ($\pi(t, T) > 0$)
  - the consumers' desire to cover their positions 'outweighs' that of the producers.
- Negative market risk premium ($\pi(t, T) < 0$)
  - the producers desire to hedge their positions outweighs that of the consumers.

Representative agents, price dynamics and forward price bounds

- Describe producers' and consumers' preferences via the utility function of two representative agents.
- First determine the forward price that makes the agents indifferent between the forward and spot market
- Second discuss how the relative willingness of producers and consumers to hedge their exposures determines market clearing prices.
- An exponential utility function parameterized by the risk aversion constant $\gamma > 0$ is:

$$U(x) = 1 - exp(-\gamma x).$$

- $\gamma := \gamma_p$ for producer and $\gamma := \gamma_c$ for consumer.
- Want to derive bounds for forward prices through the principle of certainty equivalence between two markets.

Producers and consumers forward price bounds

Following Lucia and Schwartz (2002) and Benth, Kallsen and Meyer-Brandis(2007), we assuming the electricity spot price follows a mean-reverting multi-factor additive process

$$S(t) = \Lambda(t) + \sum_{i=1}^{m} X_i(t) + \sum_{j=1}^{n} Y_j(t)$$

- $\Lambda(t)$ is the deterministic seasonal spot price level
- $X_i(t)$ is the solution to the stochastic differential equation

$$dX_i(t) = -\alpha_i X_i(t)dt + \sigma_i dB_i(t)$$

- $Y_j(t)$ is the solution to the stochastic differential equation

$$dY_j(t) = -\beta_j Y_j(t)dt + dL_j(t)$$

- $B_i(t)$ are standard independent Brownian motions and $L_j(t)$ are independent Lévy processes.

Producers and consumers forward price bounds

Proposition: The price for which the producer is indifferent between the forward and spot market is given by:

$$
\begin{aligned}
F_{pr}(t, T_1, T_2) = & \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} \Lambda(u) du + \sum_{i=1}^{m} \frac{\bar{\alpha}_i(t, T_1, T_2)}{T_2 - T_1} X_i(t) \\
& + \sum_{j=1}^{n} \frac{\bar{\beta}_j(t, T_1, T_2)}{T_2 - T_1} Y_j(t) \\
& - \frac{\gamma_p}{2(T_2 - T_1)} \int_t^{T_2} \sum_{i=1}^{m} \sigma_i^2(s) \bar{\alpha}_i^2(s, T_1, T_2) ds \\
& - \frac{1}{\gamma_p} \frac{1}{T_2 - T_1} \int_t^{T_2} \sum_{j=1}^{n} \phi_j(-\gamma_p \bar{\beta}_j(s, T_1, T_2)) ds
\end{aligned}
$$

where $\bar{\alpha}_i$ and $\bar{\beta}_j$ are defined on next page.

Producers and consumers forward price bounds

$$\bar{\alpha}_i(t, T_1, T_2) = \begin{cases} \frac{1}{\alpha_i}(e^{-\alpha_i(T_1-s)} - e^{-\alpha_i(T_2-s)}) & s \leq T_1 \\ \frac{1}{\alpha_i}(1 - e^{-\alpha_i(T_2-s)}) & s \geq T_1 \end{cases}$$

$$\bar{\beta}_j(t, T_1, T_2) = \begin{cases} \frac{1}{\beta_j}(e^{-\beta_j(T_1-s)} - e^{-\beta_j(T_2-s)}) & s \leq T_1 \\ \frac{1}{\beta_j}(1 - e^{-\beta_j(T_2-s)}) & s \geq T_1 \end{cases}$$

Producers and consumers forward price bounds

Proposition: The price that makes the consumer indifference between the forward and the spot market is given by:

$$
\begin{aligned}
F_c(t, T_1, T_2) = {} & \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} \Lambda(u) du + \sum_{i=1}^{m} \frac{\bar{\alpha}_i(t, T_1, T_2)}{T_2 - T_1} X_i(t) \\
& + \sum_{j=1}^{n} \frac{\bar{\beta}_j(t, T_1, T_2)}{T_2 - T_1} Y_j(t) \\
& + \frac{\gamma_c}{2(T_2 - T_1)} \int_t^{T_2} \sum_{i=1}^{m} \sigma_i^2(s) \bar{\alpha}_i^2(s, T_1, T_2) ds \\
& + \frac{1}{\gamma_c} \frac{1}{T_2 - T_1} \int_t^{T_2} \sum_{j=1}^{n} \phi_j(\gamma_c \bar{\beta}_j(s, T_1, T_2)) ds
\end{aligned}
$$

Market Risk Premium

- Producer prefers to sell his production in the forward market as long as the market forward price $F(t, T_1, T_2)$ is higher than $F_{pr}(t, T_1, T_2)$.
- Consumer prefers the spot market if the market forward price is mroe expensive than his indifference price $F_c(t, T_1, T_2)$.
- Thus we have the relationship:

$$F_{pr}(t, T_1, T_2) \leq F(t, T_1, T_2) \leq F_c(t, T_1, T_2)$$

Producers and consumers forward price bounds

Proposition: It holds

$$\lim_{\gamma_{p,c}\to 0} F_{pr,c}(t,T_1,T_2) = \mathbb{E}^P\Big[\frac{1}{T_2-T_1}\int_{T_1}^{T_2} s(u)du|\mathcal{F}_t\Big]$$
$$= \frac{1}{T_2-T_1}\int_{T_1}^{T_2}\Lambda(u)du + \sum_{i=1}^{m}\frac{\bar{\alpha}_i(t,T_1,T_2)}{T_2-T_1}X_i(t)$$
$$+ \sum_{j=1}^{n}\frac{\bar{\beta}_j(t,T_1,T_2)}{T_2-T_1}Y_j(t)$$
$$+ \sum_{j=1}^{m}\frac{\phi_j'(0)}{T_2-T_1}\int_t^{T_2}\bar{\beta}_j(s,T_1,T_2)ds$$

Producers and consumers forward price bounds

Moreover, we find that

$$F_{pr}(t, T_1, T_2) \leq \mathbb{E}^P \left[\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} s(u) du | S(t) \right] \leq F_c(t, T_1, T_2).$$

Forward price and the market power

- Introduce the deterministic funcion $p(t, T_1, T_2) \in [0, 1]$ describing the market power of the representative producer which therefore depends on time t and delivery period.
  - $\mathrm{p(t,\ T_1,}\ T_2) = 1 \Rightarrow$ producer has full market power
    - ★ can charge the maximum price possible in the forward market
  - $\mathrm{p(t,\ T_1,}\ T_2) = 0 \Rightarrow$ consumer has full market power
    - ★ can drive the forward price as far down as possible, equal to $F_{pr}(t, T_1, T_2)$
- The forward price $F^p(t, T_1, T_2)$ is then can be defined to be:

$$F^p(t, T_1, T_2) = p(t, T_1, T_2)F_c(t, T_1, T_2) + (1 - p(t, T_1, T_2))F_{pr}(t, T_1, T_2)$$

Forward price and the market power

Proposition: The forward price dynamics are given by

$$F^p(t,T_1,T_2) = \frac{1}{T_2-T_1}\int_{T_1}^{T_2}\Lambda(u)du + \sum_{i=1}^{m}\frac{\bar{\alpha}_i(t,T_1,T_2)}{T_2-T_1}X_i(t)$$

$$+\sum_{j=1}^{n}\frac{\bar{\beta}_j(t,T_1,T_2)}{T_2-T_1}Y_j(t)$$

$$+\frac{p(t,T_1,T_2)(\gamma_{pr}+\gamma_c)-\gamma_{pr}}{2(T_2-T_1)}\int_t^{T_2}\sum_{i=1}^{m}\sigma_i^2(s)\bar{\alpha}_i^2(s,T_1,T_2)ds$$

$$+\frac{p(t,T_1,T_2)}{\gamma_c(T_2-T_1)}\int_t^{T_2}\sum_{j=1}^{n}\phi_j(\gamma_c\bar{\beta}_j(s,T_1,T_2))ds$$

$$-\frac{1-p(t,T_1,T_2)}{\gamma_{pr}(T_2-T_1)}\int_t^{T_2}\sum_{j=1}^{n}\phi_j(-\gamma_c\bar{\beta}_j(s,T_1,T_2))ds$$

An example with constant market power and Poisson jumps

- Consider 52 contracts with weekly delivery.
- Market power suppose to be constant $p(t, T_1, T_2) = p \in [0, 1]$.
- Spot model has $m = 52$ diffusion components $X_i(t)$, and one $(n = 1)$ jump component $Y(t)$.
- Seasonal function is

$$\Lambda(t) = 150 + 20cos(2\pi/365)$$

- Mean reversion parameters for the diffusion components are $\alpha_i = 0.067/i$, with volatility $\sigma_i = 0.3/\sqrt{(}i)$, for $i = 1, 2, ...52$.
- Mimic here a sequence of mean-reverting processes with decreasing speeds of mean reversion and with decreasing volatility.

An example with constant market power and Poisson jumps

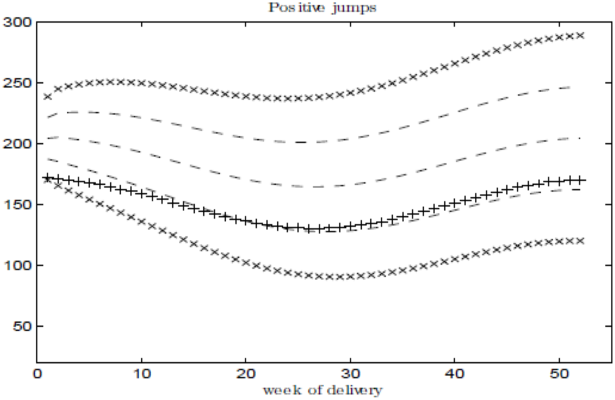

Figure 1: The indifference price curves together with the forward curves for market powers equal to $p = 0.25, 0.5$ and $p = 0.75$, in increasing order. The forecasted curve is depicted '+'. The jumps are positive of size 10

An example with constant market power and Poisson jumps

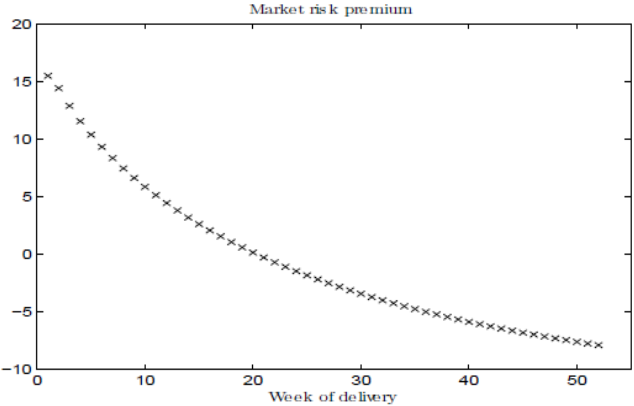

Figure 2: The market risk premium given by the difference of the forward curve with market power 0.25 and the forecasted curve

An example with constant market power and Poisson jumps

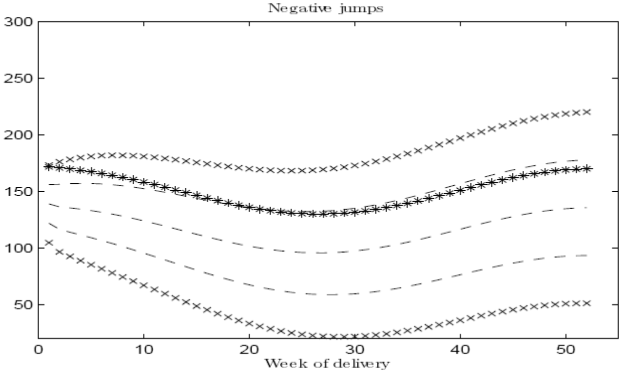

Figure 3: The indifference price curves together with the forward curves for market powers equal to $p = 0.25, 0.5$ and $p = 0.75$, in increasing order. The forecasted curve is depicted with '*'. The jumps are negative of size 10

An example with constant market power and Poisson jumps

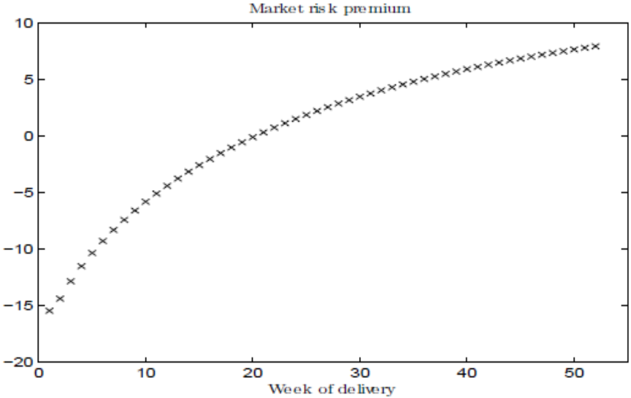

Figure 4: The market risk premium given by the difference of the forward curve with market power 0.75 and the forecasted curve

Empirical evidence: the German market

- Assume a simple two-factor spot model with jump component

$$S(t) = \Lambda(t) + X(t) + Y(t)$$

- $\Lambda(t)$ is the seasonal function

$$dX(t) = -\alpha X(t)dt + \sigma dB(t)$$

$$dY(t) = -\beta Y(t)dt + dL(t)$$

where $\alpha \geq 0$, $\beta \geq 0$, $\sigma \geq 0$, $B(t)$ is a standard Brownian motion and

$$L(t) = \sum_{i}^{N(t)} J_i$$

is a compound Poisson process.

Empirical evidence: the German market

- $N(t)$ is a homogeneous Poisson process with intensity λ and $J_i^{\prime s}$ are i.i.d. with exponential density function

$$f(j) = p\lambda_1 e^{-\lambda_1 j} I_{j>0} + (1-p)\lambda_2 e^{-\lambda_2 |j|} I_{j<0}$$

  where $\lambda_1 > 0$, $\lambda_2 > 0$are responsible for the decay of the tails for the distribution of positive and negative jump sizes and I is the indicator function.
- Assume that $N(t)$, J and $B(t)$ are independent.

Empirical evidence: the German market

- For seasonal component we assume

$$\Lambda(t) = a_0 + a_1 I_{t=su} + a_2 I_{t=Mo} + a_3 I_{t=Tu,We,Th} + a_4 I_{t=Sa} + a_5 cos\left[\frac{6\pi}{365}(t + a_6)\right] + a_7 t$$

Empirical evidence: the German market

a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	Squared Error
19.43	-11.43	0.78	2.13	-6.13	1.2	53.64	0.016	39420.74

Table 1: Estimated coefficients of $\Lambda(t)$

α	σ	p	λ	λ_1	λ_2	β
0.44	5.2	0.72	0.054	0.031	0.053	0.2

Table 2: Parameter estimates for OU and Jump components

Empirical evidence: the German market

	Mean	Std. Dev.	Skewness	Kurtosis
Empirical $S(t)$	31.6	15.2	2.7	14.5
Simulated $S(t)$	32.1	15.6	2.2	13.8

Table 3: Empirical ad simulated moments (1000 paths)

Empirical evidence: the German market

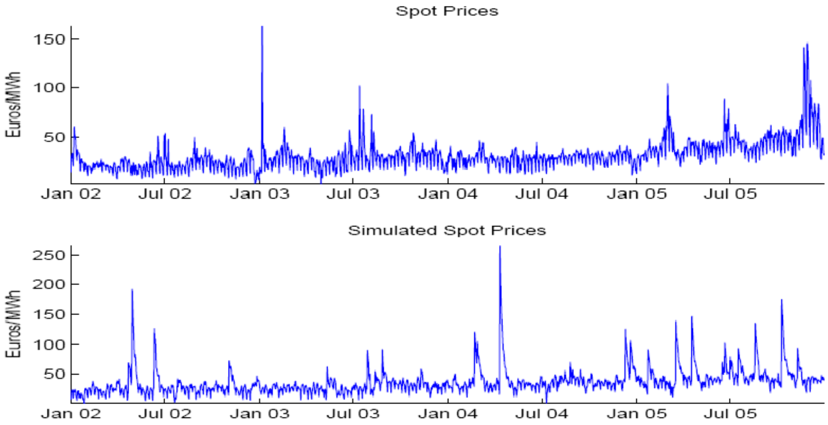

Figure 5: Spot and Simulated Spot Prices

Empirical evidence: the German market

- In order to calculate the market power $p(t, T_1, T_2)$ and the forward premium $\pi(t, T_1, T_2)$
  - Need to choose risk aversion coefficients γ_c and γ_{pr}
  - $F_c(t, T_1, T_2)$ and $F_{pr}(t, T_1, T_2)$ depends on γ_c and γ_{pr}
  - Minimizing the distance between $F_c(t, T_1, T_2)$, $F_{pr}(t, T_1, T_2)$ and the market prices of forwards $F(t, T_1, T_2)$ repectively.

Empirical evidence: the German market

- Time range: $t = 1 = 02/Jan/2002$ until $t = 1461 = 31/Dec/2005$
- We have prices for forward contracts with delivery one month, quarterly and yearly.
- As long as there is a price on day t, we can determine all values for γ_{pr} and γ_c such that

$$F_{pr}(t, T_1, T_2) \leq F(t, T_1, T_2) \leq F_c(t, T_1, T_2)$$

- This will gives us the intervals I_{pr}^t and I_c^t containing the values for γ_{pr} and γ_c

Empirical evidence: the German market

- Algorithm guarantees that no forward prices $F(t, T_1, T_2)$ will lay out the bounds
- Results: $\gamma_{pr} \in [0.421, \infty]$ and $\gamma_c \in [0.701, \infty]$
- In our calculation, we pick $\gamma_{pr} = 0.421$ and $\gamma_c = 0.701$
  - producers are less risk-averse than consumers.
- Split data in three non-overlapping periods.
  - Key criterion is to include in the first period all contracts that were being traded on January $2, 2002$
  - The second period starts when all contracts that were being traded on January $2, 2002$ are no longer traded

Empirical evidence: the German market

t	Type	# Contracts	Delivery Periods	$F(t, T_1, T_2)$
01/Jan/2002	monthly	18	Jan 2002 - May 2003	$F(2, T_1, T_2)$
01/Jan/2002	quarterly	7	2nd qtr 2002 - 4th qtr 2003	$F(2, T_1, T_2)$
01/Jan/2002	yearly	3	2003 - 2005	$F(2, T_1, T_2)$
03/Mar/2003	monthly	7	Feb 2003 - Aug 2003	$F(400, T_1, T_2)$
03/Mar/2003	quarterly	7	2nd qtr 2003 - 4th qtr 2004	$F(400, T_1, T_2)$
03/Mar/2003	yearly	3	2004 - 2006	$F(400, T_1, T_2)$
04/Oct/2005	monthly	7	Oct 2005 - Apr 2006	$F(1373, T_1, T_2)$
04/Oct/2005	quarterly	7	1st qtr 2006 - 3rd qtr 2007	$F(1373, T_1, T_2)$
04/Oct/2005	yearly	6	2006 - 2011	$F(1373, T_1, T_2)$

Table 4: Forward contracts

Empirical evidence: the German market

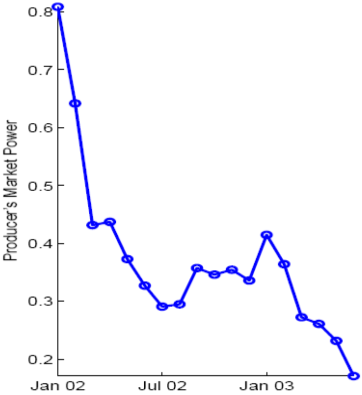

Figure 6: Producer's market power and market risk premium, 18 monthly contracts with $t =$ January 2 2002

Empirical evidence: the German market

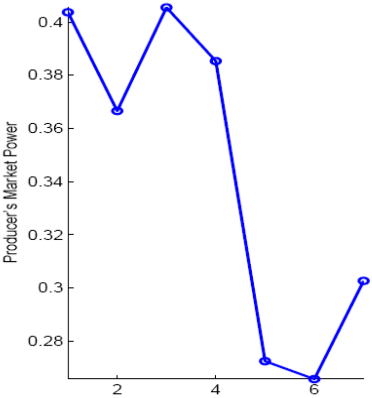

Figure 7: Producer's market power and market risk premium, 7 quarterly contracts with $t =$ second quarter 2002

Empirical evidence: the German market

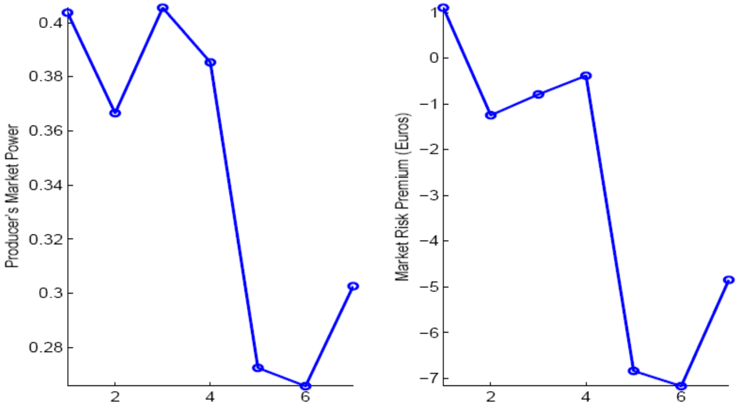

Figure 8: Producer's market power and market risk premium, 3 yearly contracts with $t = 2002$

Conclusions and Open Problems

- Provide a framework that allows us to explain how risk preferences of market players explain the sign and magnitude of the market risk premium arocss different forward contract maturities.
- An attainable set where consumers and producers are willing to strike a deal.
- Apply the model to German electricity market, empirical results endorse the theoretical predictions.
- Observed that each class of contract(monthly quarterly or yearly), the producer's market power and the market risk premium show a term structure that is decreasing as time to maturity of the forward contract increases.

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Thank you!