

Time Scale Decomposition of Economic Relationships Using Wavelet Analysis

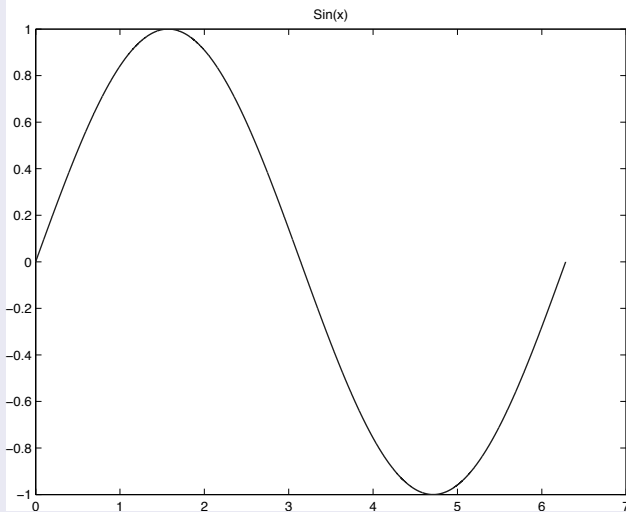
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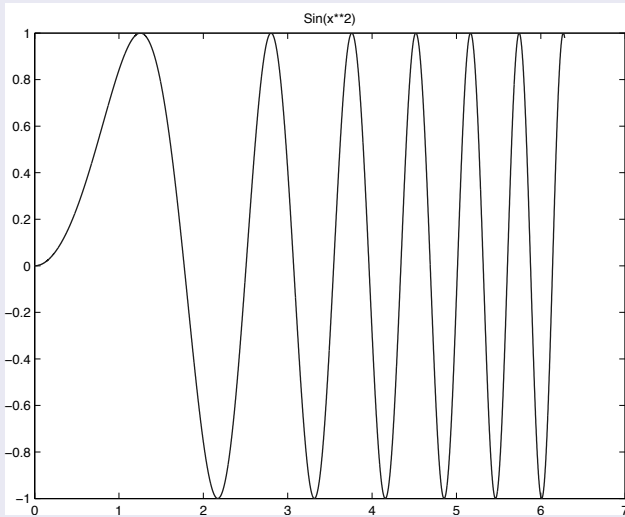
April 9, 2007

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 - Qualitative Overview of Wavelet Analysis
 - Wavelets and the Analysis of Economic and Financial Data
- 2 Results of Ramsey and Lampart's Investigations
 - Consumption and Income
 - Synopsis of Empirical Findings
- 3 My Research Plans
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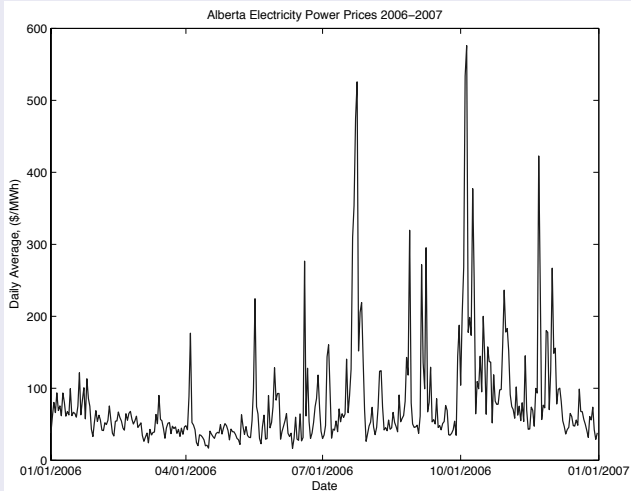
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A not so Simple Example



What are Wavelets?

- Defining quality: A wavelet is a function defined on \mathbb{R} which, when subjected to translations (by integers) and dyadic dilations yields an orthonormal basis of $L^2(\mathbb{R})$.

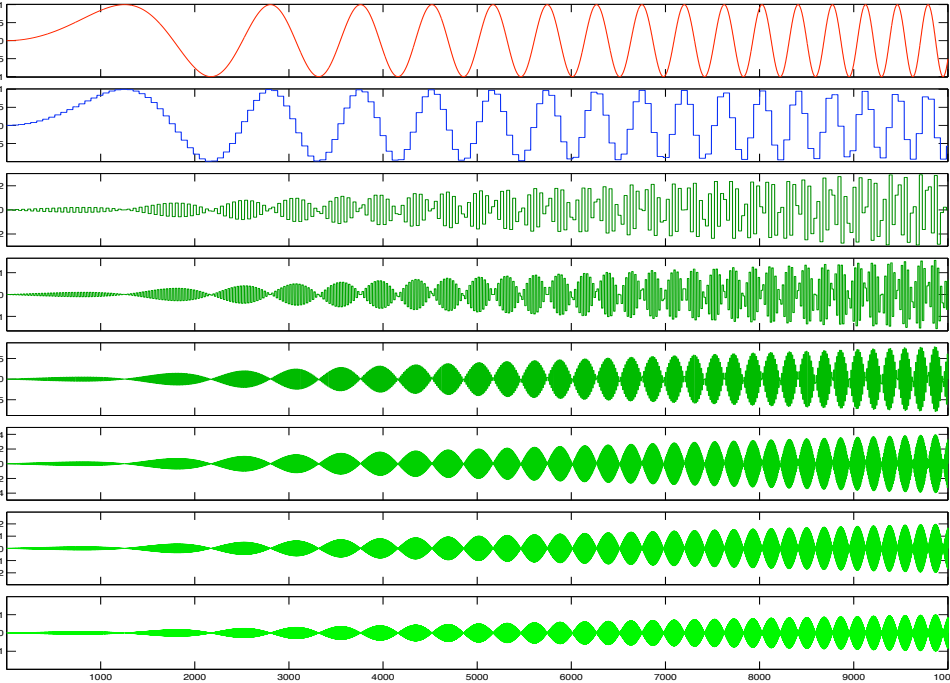
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- On the next slide I present the 6 scale decomposition of $\sin(x^2)$ by Haar wavelets.

Decomposition at level 6 : $s = a_6 + d_6 + d_5 + d_4 + d_3 + d_2 + d_1$.



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 - ② At 'low' frequencies the wavelet has a large time support allowing it to identify long periodic behavior.

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 - 3 Really short view (hours), interested in ephemeral changes in market behavior.

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- Traditional economics views the consumption/income relationship as the two fold distinction between permanent and transitory income.
- However, it may be that the relationship between consumption and income may depend on a *range* of time scales, not just 2.

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 - ② The decomposition indicated that the delay in the relationship between two variables may be a function of the state space of the dynamical system.

The Methodology of Their Analysis pg 1 of 2

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 - ③ In general, $S_{j-1} = S_j + D_j$
- The multiresolution *decomposition* of a signal is, $\{S_J, D_J, D_{J-1}, \dots, D_j, \dots, D_1\}$.

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- Instead of looking at the time relationship between consumption and income averaged over all time scales, they examined the relationship on *each* time scale *separately*.

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- The price models for electricity are considerably richer in structure than for other commodities.
- Conventional risk management techniques are of limited value in electricity markets.
- While my project focuses on electricity prices, it is my feeling the methods will be equally valuable when applied to other commodities, e.g., natural gas or oil.

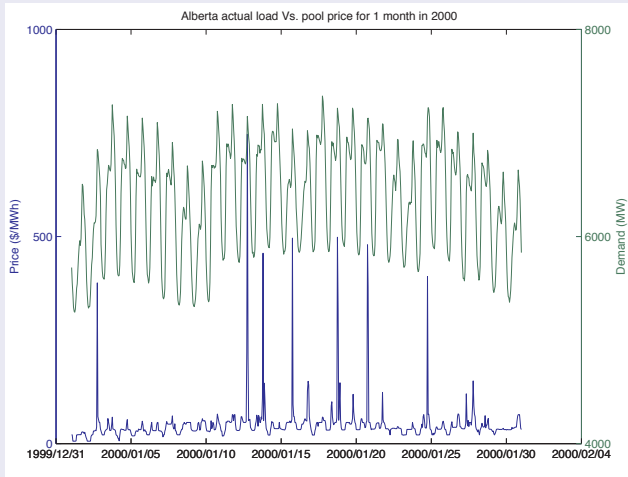
What I Want to do

- My plan is to use wavelets to provide insight into the functional relationship between demand and price in Alberta's electricity market by decomposing each variable by scale and examine the relationship at each time scale separately.

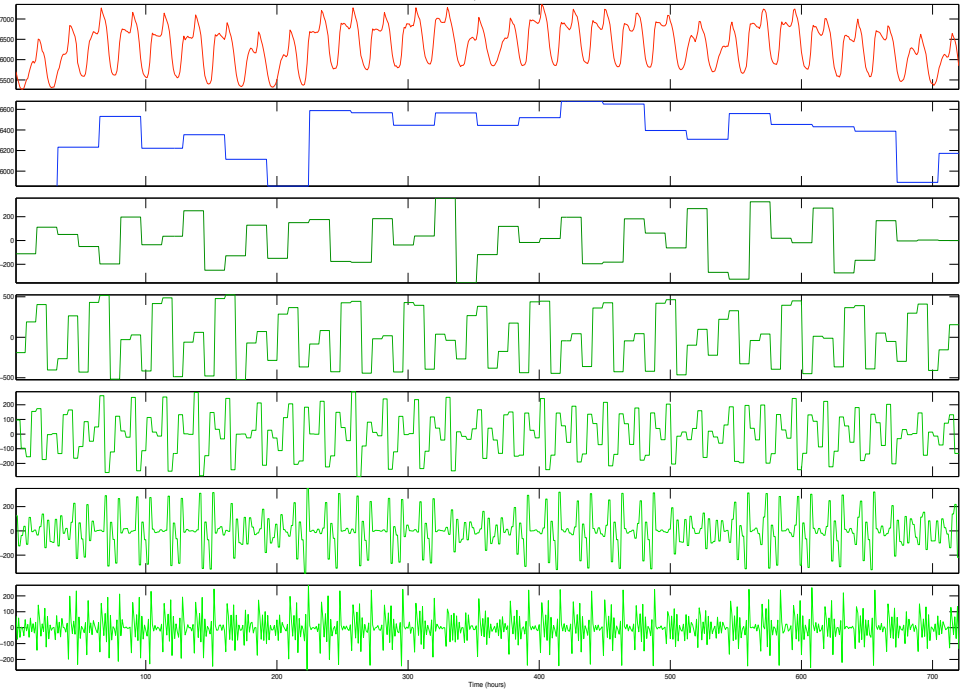
What I Want to do

- My plan is to use wavelets to provide insight into the functional relationship between demand and price in Alberta's electricity market by decomposing each variable by scale and examine the relationship at each time scale separately.
- It is hoped that this analysis will provide valuable information that can be leveraged when constructing forecasting models.

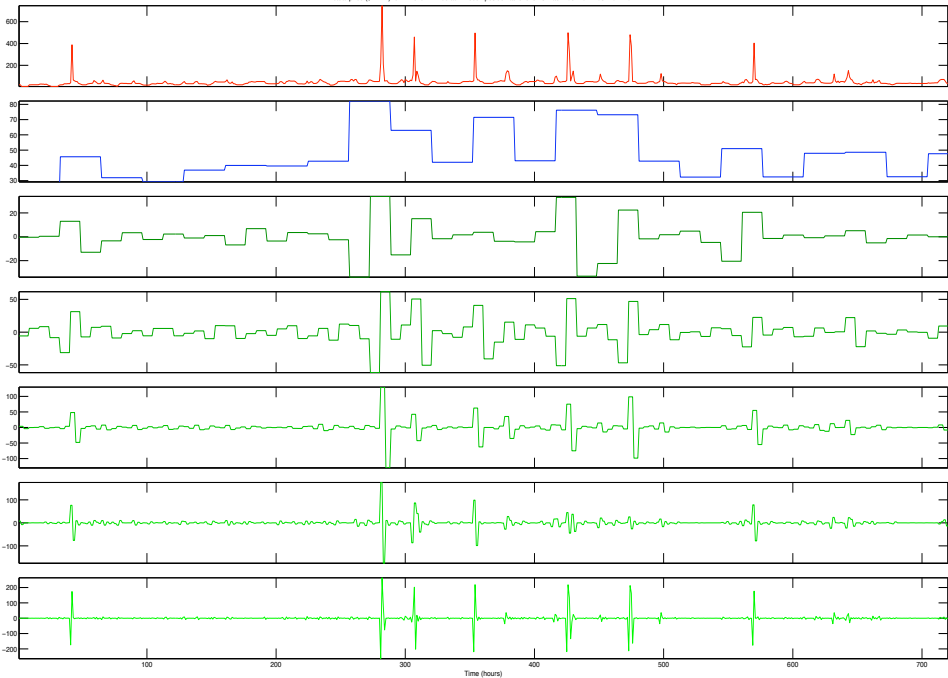
Price and Demand Plot



Actual load for 1 month in Alberta - Decomposition at level 5 - $s = a5 + d4 + d3 + d2 + d1$



Actual price (\$/MWh) for 1 month in Alberta - Decomposition at level 5 : $s = a5 + d5 + d4 + d3 + d2 + d1$.



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- Use wavelet methods to transform the data and feed it to the input neurons in an artificial neural network.
- Their algorithm accurately predicts one-hour ahead load and pool-price data based on historical data.
- A fun summer project would be to wrap their algorithm into a graphical interface suitable for use on the trading floor.

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- By decomposing economic and financial variables by time-scale, fundamental insights into the relationships between the variables can be made.