Portfolio Optimization Subject to Market Risk Constraints

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Motivations

Liberalization of energy market:

- new set of problems for power companies,
- instrument for hedging:
  - gap between production and demand,
  - financial risk.

Objectives for Électricité de France: integrate risk constraints in the historical problem which consisted in managing the power plant generation at lowest expected cost.

Outline of the presentation

We consider the maximization of expected profit $\mathbb{E}\big[J(\mathrm{a},\xi)\big]$ subject to risk constraint

$$\begin{cases} \sup_{\mathrm{a}\in\mathbb{A}} \mathbb{E}\big[J(\mathrm{a},\xi)\big] \\ \text{s.t. } \mathcal{R}\big(-J(\mathrm{a},\xi)\big) \le \gamma, \end{cases}$$

where

- $\mathbb{A} \subset \mathbb{R}^n$ is a set of decisions;
- ξ is a random variable defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$;
- J is a mapping, such that for any decision $\mathrm{a} \in \mathbb{A}$, the random variable $J(\mathrm{a},\xi)$ represents the profit of the decision maker (DM);
- $\mathcal{R}$ is the risk measure on the loss $L(\mathrm{a},\xi) := -J(\mathrm{a},\xi)$, associated with a level constraint $\gamma \in \mathbb{R}$.

Expected utility and extension

In these models we compare:

- Lotteries
  - Expected utility model: $\mathcal{V}(\mu) = \int_{\mathbb{R}} U \mathrm{d}\mu$,
  - Maxmin à la Maccheroni: $\mathcal{V}(\mu) = \min_{U \in \mathcal{U}} \int_{\mathbb{R}} U \mathrm{d}\mu$.
- Random variables
  - *RDEU*: $\mathcal{V}(X) = -\int_{-\infty}^{+\infty} U(x) \mathrm{d}\varphi\Big(\mathbb{P}(X > x)\Big)$,
  - Utility à la Choquet: $\mathcal{V}(X) = \int_{Ch} U(X) \mathrm{d}\nu$,
  - Multi-prior model: $\mathcal{V}(X) = \min_{\mathbb{P} \in \mathcal{P}} \int U(X) \mathrm{d}\mathbb{P}$.

Expected utility or extensions

Consider the maximization of the expected utility of the profit $J(a, \xi)$

$$\sup_{\mathbf{a} \in \mathbb{A}} \mathbb{E}\big[U\big(J(a, \xi)\big)\big],$$

where U is a utility function.

The function U captures more or less risk aversion of the DM.

Extensions: non-expected utility theories.

The infimum of expectations class of risk measures

Let us consider the class of risk measures expressed by

$$\mathcal{R}_\rho(L) := \inf_{\eta \in \mathbb{R}} \mathbb{E}\big[\rho\big(L,\eta\big)\big] \quad \text{for} \quad L \in L_\rho(\Omega, \mathcal{F}, \mathbb{P}).$$

Assumption

- *The function* $\rho \colon \mathbb{R} \times \mathbb{R} \to \mathbb{R}$.
- *The set* $L_\rho(\Omega, \mathcal{F}, \mathbb{P})$ *of random variables is such that*
  - $\rho\big(L,\eta\big)$ *is integrable for any* $\eta \in \mathbb{R}$,
  - $\mathcal{R}_\rho(L) > -\infty$.

Two equivalents formulations (1/2)

The following two hypothesis are admitted.

Hypothesis

H1. *The function* $\eta \mapsto \rho(x, \eta)$ *is convex.*

H2. *For* $L \in L_\rho(\Omega, \mathcal{F}, \mathbb{P})$*, the function* $\eta \mapsto \mathbb{E}\big[\rho\big(L, \eta\big)\big]$ *is continuous and tends to* $+\infty$ *when* $\eta \to +\infty$*.*

Two equivalents formulations (2/2)

Result

Under H1. and H2., the maximization problem subject to risk constraint

$$\sup_{a \in \mathbb{A}} \mathbb{E}\big[J(a,\xi)\big] \ s.t. \ \mathcal{R}_{\rho}\big(-J(a,\xi)\big) \leq \gamma \qquad (1)$$

is equivalent to the max min *expected utility problem*

$$\sup_{(a,\eta)\in \mathbb{A}\times\mathbb{R}} \ \inf_{U \in \mathcal{U}} \mathbb{E}\left[U\big(J(a,\xi),\eta\big)\right]$$

where the set of utility functions $\mathcal{U}$ *is defined by:*

$$\mathcal{U} := \left\{ U^{(\lambda)} : \mathbb{R}^2 \to \mathbb{R}\,,\ \lambda \geq 0 \mid U^{(\lambda)}(x,\eta) = x + \lambda\big(-\rho(-x,\eta)+\gamma\big) \right\}.$$

Conditional Value-at-Risk and loss aversion

When the risk measure $\mathcal{R}_\rho$ is the CVaR, we have

$$U^{(\lambda)}(x,\eta) = x - \frac{\lambda}{1-p}(-x-\eta)_+ - \lambda\eta + \lambda\gamma \,. \qquad (2)$$

1. We consider only the argument x (profit) and we interpret η as a parameter.
2. Adding a constant gives the utility function

$$x+\eta-\frac{\lambda}{1-p}(-x-\eta)_+ = \begin{cases} x+\eta & \text{if} \quad x+\eta \geq 0\,, \\ (1+\frac{\lambda}{1-p})(x+\eta) & \text{if} \quad x+\eta < 0\,. \end{cases}$$

Loss aversion interpretation

- We interpret the ratio of derivatives

$$\theta := 1 + \frac{\lambda}{1-p}$$

as a *loss aversion* parameter, introduced by Kahneman and Tversky.

- The utility function

$$U(x) = x + \eta + (1-\theta)(-x-\eta)_+$$

is parameterized by

  - *reference point* $-\eta$,
  - *loss aversion* θ

and expresses the property that one monetary unit more than the anchorage gives one unit of utility, while one unit less gives $-\theta$.

Portfolio Optimization Problem

Suppose that a DM splits its investment between a risk free asset ξ^0 (deterministic[2]) in proportion $\mathrm{a} \in [0,1]$ and a risky asset ξ^1 (Gaussian distribution $\mathcal{N}(M,\Sigma)$[3]) in proportion $1-\mathrm{a}$. Thus, the value of the portfolio is

$$J(\mathrm{a},\xi) = \mathrm{a}\xi^0 + (1-\mathrm{a})\xi^1 = \mu(\mathrm{a}) + \sigma(\mathrm{a})N, \quad N \sim \mathcal{N}(0,1) \qquad (3)$$

with

$$\mu(\mathrm{a}) = \mathrm{a}\xi^0 + (1-\mathrm{a})M \quad \text{and} \quad \sigma(\mathrm{a}) = (1-\mathrm{a})\Sigma\,.$$

[2] Numerical value of 1 030 US dollar, corresponds to a risk-free rate of return of 3%.

[3] We assume that the risky asset follows a Normal distribution where the mean and standard deviation are calibrated based on the annual MSCI world developed market performance index between 1970 and 2009. Hence the Mean M=1 113.3425 US dollar and the standard deviation Σ=186.29 US dollar.

Confidence level and loss aversion (1/2)

The corresponding loss aversion coefficient is $\theta^\sharp = 1 + \frac{\lambda^\sharp}{1-p}$ (notice that $\theta^\sharp$ does not depend on γ, because $\lambda^\sharp$ does not). Figure 1 represents the curve $p \mapsto \theta^\sharp$ of loss aversion coefficient $\theta^\sharp$ as a function of confidence level p.

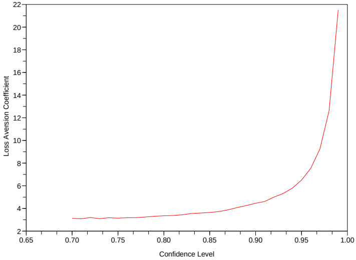

Figure: Confidence level and the corresponding loss aversion parameter

Confidence level and loss aversion (2/2)

We observe

- high values for loss aversion, well above the empirical findings (median value of 2.25 in Tversky-Kahneman, 1992), for $p \geq 0.9$;
- a numerical computation reveals that a loss aversion parameter of 3.07 corresponds to confidence level $p = 0.70$.

Conclusion: Figure 1 suggests that controlling this portfolio Conditional Value-at-Risk at lower confidence levels between 0.7 and 0.8 reveals a quite acceptable loss aversion slightly higher than 3.

Conclusion

- Theoretical results:
  - Formulations of stochastic optimization problem under risk constraints,
  - CVaR constraint associated to loss aversion utility functions.

  Ref: B. Seck, L. Andrieu and M. De Lara: Parametric Multi-Attribute Utility Functions for Portfolio Optimization Subject to Market Risk Constraints, Journal of Theory and Decision, Vol. 72, No. 2, 257-271, 2012.

- Application: Taking risk into account in power portfolio management (EDF case)
  - Stochastic dynamic programming,
  - Approximation by utility function.

  Ref: B. Seck, L. Andrieu and M. De Lara: Taking Risk into Account in Electricity Portfolio Management, in Handbook of Power Systems II, Part 4, 433-448. Springer, 2009.