A 'Simple' Hybrid Model for Power Derivatives

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Outline

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  - Some problems
- Our Model
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  - The supply model
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  - The mean reverting model
  - Finding the market price of risk
- Some Results
  - The parameters
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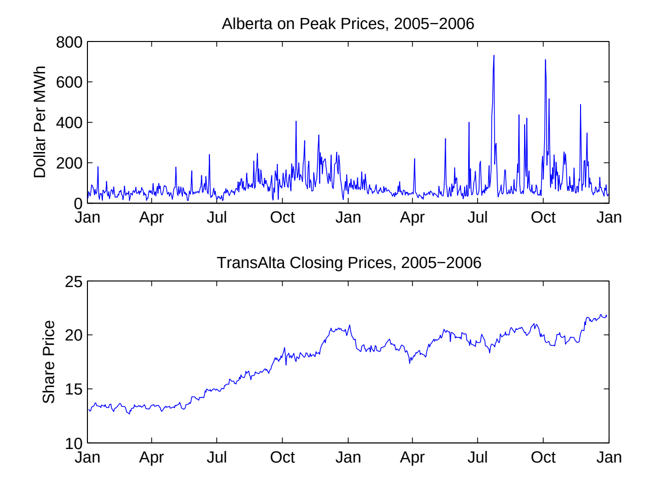
Look at that ‘volatility’



Some traditional models: Reduced form

- Black-Scholes World: $\frac{dS}{S} = \mu dt + \sigma dW$
  $\lim_{t\to\infty} E[S] \to \infty$, $Var[S] \to \infty$
- Mean-Reverting World: $d \log S = \kappa(\mu - \log S)dt + \sigma dW$
  $\lim_{t\to\infty} E[logS] \to \mu$, $Var[\log S] \to \sigma^2/(2\kappa)$
- These two are relatively simple models, but they fail to capture many of the characteristics exhibited in power markets. So we increase the complexity.

Some traditional models: Reduced form

- Mean-Reverting Jump Diffusion:
  $d \log S = \kappa(\mu - \log S)dt + \sigma dW + \beta dJ$
- We continue in this fashion, adding stochastic volatility and or stochastic means etc.
- We soon face issues with calibration and mathematical tractability
- so what to do?

Hybrid Models: two examples

- Barlow, Math Finance, 2002 :

$$S(t) = \begin{cases} \left(\frac{a_0 - D_t}{b_o}\right)^{(1/\alpha)} & , D_t < a_0 - \epsilon_0 b_0 \\ \epsilon_0^{(1/\alpha)} & , D_t \geq a_0 - \epsilon_0 b_0 \end{cases}$$

  - a_0 is the max supply capacity, $\alpha < 1$
  - D_t is demand and follows an OU process, b_0 is a constant
  - simulation results seem promising, but no closed form derivatives price

Hybrid Models: two examples

- Davison et. al., Power Engineering Review IEEE, 2002 :

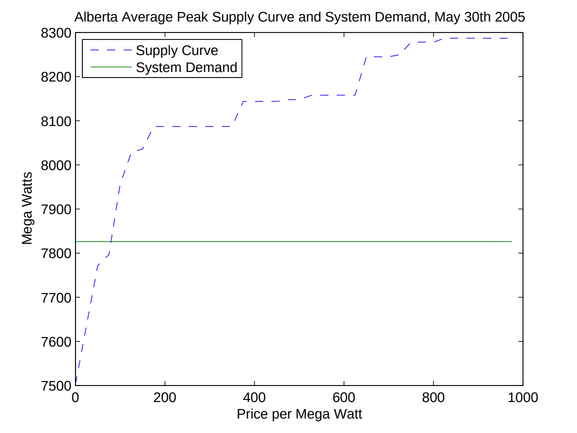
$$S(t) \leftarrow P(t) = (1 - \epsilon(\alpha(t)))P_L(t) + \epsilon(\alpha(t))P_H(t)$$

  - P_L is a low price distribution, P_H is a high price distribution
  - ϵ is a switching parameter, α is the ratio of power demand to capacity
  - $\lim_{\alpha \to 0} \epsilon = 0$, $\lim_{\alpha \to 1} \epsilon = 1$,
  - simulation results seem promising, but no closed form derivatives price

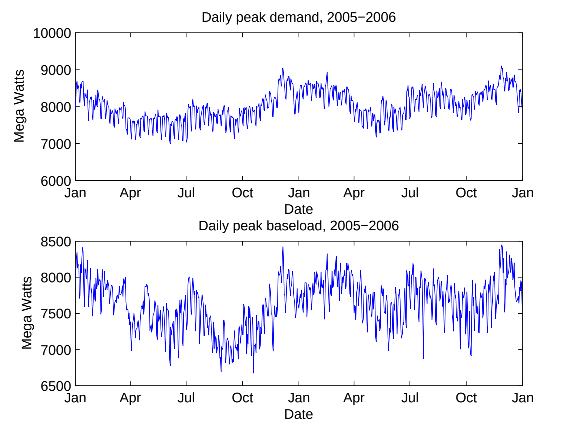
The problems, but

- The reduced form models
  - Become complex and are difficult to calibrate.
  - Time consuming and non-intuitive to use.
  - Difficult to obtain closed form solution for options.
- Hybrid models
  - Become very complex when more parameters are added.
  - Generally no closed form solutions for option prices.
- But we need to capture the dynamics, so these type of models are necessary.

A motivational figure



Baseload supply and demand



Supply Equals Demand

- $D(t, P) = D(t)$, demand is inelastic
- $S(t, P) = aB(t, P) + Pk(t, P) = aB(t) + Pk(P)$
  - B, is baseload supply function and is price invariant.
  - Pk, is peak supply and is time invariant.
- Equilibrium price $P = Pk^{-1}(D(t) - aB(t))$
  - $Pk' > 0$, $Pk'' < 0$, concavity requirement.
  - $D(t)$ is a mean-reverting process.
  - $B(t)$ is stochastic process.

Baseload

- Baseload form:
  - $B(t) = f(t) + S_b(t) = \text{Deterministic} + \text{Noise}$
  - $f(t) = a_1 + b_1 t + \sum_{i=1}^{N} c_i sin(2\pi\omega_i + d_i)$
  - $S_b(t) = \begin{cases} <\alpha, G_t> & , Case1 \\ \int_0^t \kappa_s(\mu_s - S_b(t))dt + \int_0^t \sigma_s dW_t & , Case2 \end{cases}$
  - $G_t = G_0 + \int_0^t AG_s ds + M_t$., G_t state space is the set of unit vectors.
  - α are weight parameters, A is the rate matrix $\Pi = e^A$

Peak load form and Demand

- Peak load form:
  - $Pk = b \log(cP + \xi)$
- Demand
  - $D(t) = f_1(t) + \hat{D}(t) =$ Deterministic + Noise
  - $f_1(t) = a'_1 + b'_1 t + \sum_{i=1}^{N} c'_i sin(2\pi\omega'_i + d'_i)$
  - $\hat{D}(t) = \int_0^t \kappa(\mu - \hat{D}(s))ds + \sigma \int_0^t dB_s$

Price

- Price
  - $$P(t) = \frac{1}{c}(\exp(-\frac{aB(t) - D(t)}{b}) - \xi) \in [-\frac{\xi}{c}, \infty). \tag{1}$$
  - $P(t)$, $B(t)$, and $D(t)$ are observable to some extent.
  - So we just need to estimate a, b, c, ξ.
  - Non linear regression can be used to obtain estimates quickly.

Derivative

The price of a claim:

- $V(t) = E[\frac{m(T)}{m(t)} g(T) | \mathcal{F}_t]$
- Electricity is non-storable (generally) so standard riskneutral or replication pricing is not possible.
- M is the discount factor, g is the claim at time $T > t \geq 0$.
- Let $\frac{m(T)}{m(t)} = e^{-(r-\gamma)(T-t)}$, where r is the riskfree rate, and γ is the market price of risk.
- $g(T) = (P(T) - K)^+$ for a call option.

So

$$V(t) = e^{-(r-\gamma)(T-t)} \int_{-\infty}^{\infty} (P - K)^+ dF(P)$$

Call price with Markov chain baseload

- The price of a call:

$$V_t = e^{-(r-\gamma)(T-t)} \langle C_t, e^{A(T-t)} G_t \rangle. \quad (2)$$

Here $C_t = (C_t^1, \ldots, C_t^N)'$ where

$$C_t^i = \frac{1}{c}(e^{-\lambda_i + \mu_z + \sigma_z^2/2} \Phi(d1) - (\xi + cK)\Phi(d2))$$

$$d1 = \frac{\mu_z + \sigma_z^2 - \lambda_i - \log(cK + \xi)}{\sigma_z}, \quad d2 = d1 - \sigma_z$$

$$\mu_z = \frac{\mu_D(T-t)}{b}, \quad \sigma_z^2 = \frac{\sigma_D(T-t)^2}{b^2}$$

$$\lambda_i = \frac{a(f(T-t) + \alpha_i)}{b} .$$

α_i is the weight parameter in state i.

Call price with mean-reverting baseload

- The price of a call:

$$V_t = \frac{e^{-(r-\gamma)(T-t)}}{c}\left[e^{\sigma_z^2/2-\mu_z}\Phi(d_1) - (\xi + Kc)\Phi(d_2)\right] . \quad (3)$$

Where,

$$d_1 = \frac{\sigma_z^2 - \mu_z - \log(\xi + Kc)}{\sigma_z}$$

$$d_2 = -\frac{\mu_z + \log(\xi + Kc)}{\sigma_z} = d_1 - \sigma_z$$

$$\mu_z = \frac{a\mu_{sm}(T-t) - \mu_D(T-t)}{b}$$

$$\sigma_z^2 = \left(\frac{\sigma_D(T-t)}{b}\right)^2 + \left(\frac{a\sigma_{sm}(T-t)}{b}\right)^2$$

Finding the Market Price of Risk

- The price of a Forward:

$$F(t) = E[\frac{m(T)}{m(t)}P(T)|\mathcal{F}_t] = e^{-(r-\gamma)(T-t)}E[P(T)|\mathcal{F}_t].$$

$$\text{Then, } \gamma = \frac{\log(F(t)) - \log(E[(P(T))|\mathcal{F}_t]) + r(T-t)}{T-t}.$$

- This can be used to price other derivatives as well.
- We can thus use our spot price model, which is based on real world data, and then link it to the 'risk-neutral' world of derivatives.

Deterministic estimation

- Deterministic Estimates:

$$f(t) = f_S(t) + f_W(t).$$

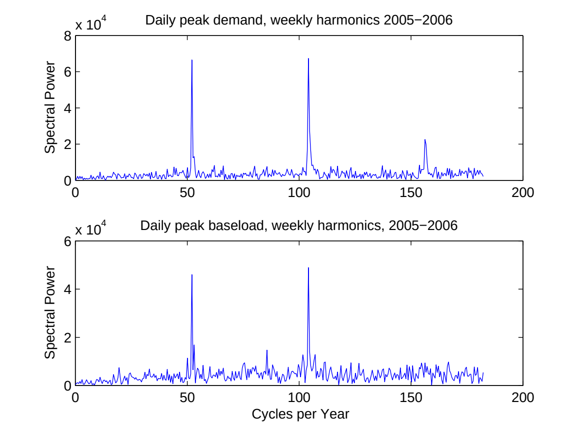
$f_S(t)$ and $f_W(t)$ are the seasonal and weekly components.

$$f_S(t) = a\sin(2\pi t/365 - t_a) + b\sin(4\pi t/365 - t_b) + ct + d$$

$$f_w = \sum_{n=1}^{N} w_n \sin(2\pi \frac{n}{365 \cdot 7} + t_{w_n})$$

- Data used in Alberta Spot price data covering years 2005-2007, with years 2005-2006 as in sample data.

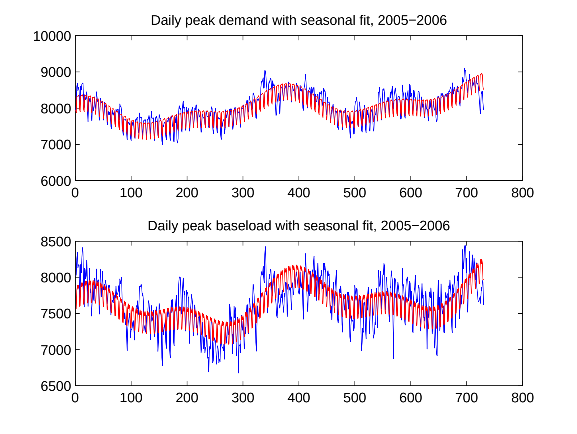
Frequency plot of weekly cycles



Deterministic estimation results

Variable	Supply	Demand
a	262.5	328.9
b	158.8	195.7
c	0.5737	0.8824
d	7427	7706
w_1	-149.3	216.8
w_2	93.62	136.9
w_3		47.78
t_a	-187.7	-10.97
t_b	-5.324	-5.324
t_{w_1}	6.544	9.612
t_{w_2}	2.011	58.51
t_{w_3}		0.0578

Deterministic fit with actual data



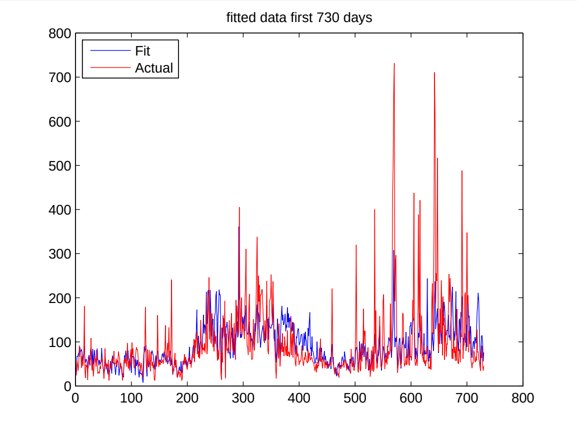
Estimating the supply parameters

The following we produced from running a nonlinear regression with MATLAB. On the Pricing equation:

$$P(t) = \frac{1}{c}(\exp(-\frac{aS_b(t) - D(t)}{b}) - \xi).$$

Variable	Estimated Value
a	1.115 ± 0.0853
b	685.89 ± 298.7774
c	$0.0049 \pm \mathbf{0.0050}$
ξ	$0.0832 \pm \mathbf{0.2130}$

The Parameters



Markov chain transition Matrix

$S_b(t) = < \alpha, G_t >$

$$\Pi = \begin{pmatrix} 0.9164 & 0.4783 & 0.6327 \\ 0.0331 & 0.5217 & 0 \\ 0.0505 & 0 & 0.3673 \end{pmatrix} = e^A$$

and

$$\alpha = (4.8954, 429.2737, -457.0059)'.$$

- 4.8954 is the average value in state 1
- 429.2737 is the average value in state 2 (1.5 stds above de-cycled mean)
- -457.0059 (1.5 stds blow de-cycled mean) is the average value in state 3.

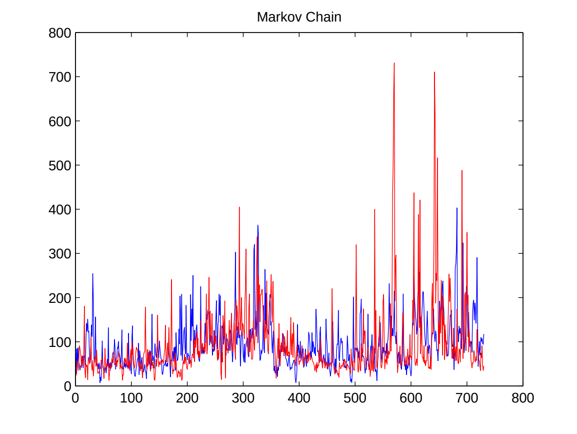
Mean-reverting estimates

$$dX(t) = \kappa(\mu - X(t))dt + \sigma dW(t)$$

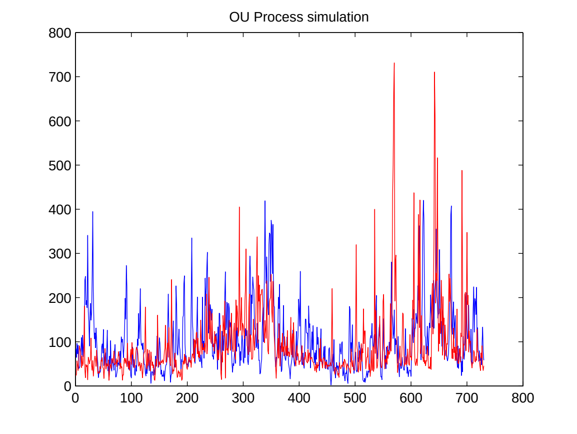
Variable	Estimated Value Supply	Estimated Value Demand
κ	0.2723	0.1371
μ	-2.0789	1.4883
σ	188.8025	133.1162

- κ is the speed of mean-reversion.
- μ is the long run mean.
- σ is the amplitude of noise.

Simulated path with Markov chain model



Simulated path with mean-reverting model



In sample and out of sample simulation results-1000 runs

	2005[1]	2006[1]	2007[1]	2005[2]	2006[2]	2007[2]
Mean	85.39	99.00	84.26	97.57	117.88	111.62
Std.	57.04	98.80	83.84	63.24	78.68	71.91
Skewness	1.77	3.51	4.80	1.90	2.52	2.05
Kurtosis	7.24	17.91	29.67	8.019	13.57	10.76
	2005[3]	2006[3]	2007[3]			
Mean	95.49	108.57	112.24			
Std.	72.91	78.30	85.17			
Skewness	2.30	2.64	2.12			
Kurtosis	11.93	15.64	11.47			

- [1] indicates market data.
- [2] indicates results from the Markov chain model.
- [3] are the results using the Mean-reverting.

Call prices for Markov chain model with $K = 92$

	T-t=90	T-t=120	T-t=210	T-t=270	T-t=365
γ=0.00	3.0792	16.6157	14.5388	73.5757	71.0398
$\gamma = 0.05$	3.1174	17.0305	14.9631	76.3480	74.6821
$\gamma = 0.10$	3.1561	17.4556	15.3998	79.2247	78.5111
$\gamma = 0.15$	3.1953	17.8914	15.8492	82.2097	82.5365
$\gamma = 0.20$	3.2349	18.3380	16.3118	85.3073	86.7682

- Notice the Higher levels of price at $T - t = 210$ and $T - t = 365$.
- Seasonality of price behavior is being picked up in the option prices.

Call prices for Mean-reverting model with $K = 92$

	T-t=90	T-t=120	T-t=210	T-t=270	T-t=365
γ=0.00	10.2942	27.5875	25.2781	83.5000	80.8677
$\gamma = 0.05$	10.4219	28.2762	26.0158	86.6462	85.0139
$\gamma = 0.10$	10.5512	28.9821	26.7751	89.9109	89.3726
$\gamma = 0.15$	10.6821	29.7056	27.5565	93.2986	93.9548
$\gamma = 0.20$	10.8146	30.4472	28.3608	96.8140	98.7720

- The mean-reverting model provides higher prices in general.
- However this can be changed with different selections of α and the number of states.

Positives

- Positives
  - The model is based on observable market data.
  - Has economic intuition
  - Calibration is easy and fast
  - Closed form solutions for options, great for time constrained analysis.
  - Seasonality of price behavior is being picked up in the option prices.
  - Can be scaled up or down in complexity-We are adding emissions uncertainty into the model.

Negatives

- Negatives
  - More complicated than the BS model
  - No upper bound on price
  - Three data sets (Price, demand, and supply) needed for calculation instead of one
  - The smooth log function for peak load cannot replicate large discontinuous jumps in supply for smaller markets like Alberta