Stochastic Modelling of Electricity and Related Markets: Chapter 1

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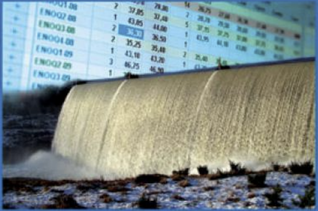
STOCHASTIC MODELLING OF ELECTRICITY AND RELATED MARKETS



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World Scientific

Scope of the text

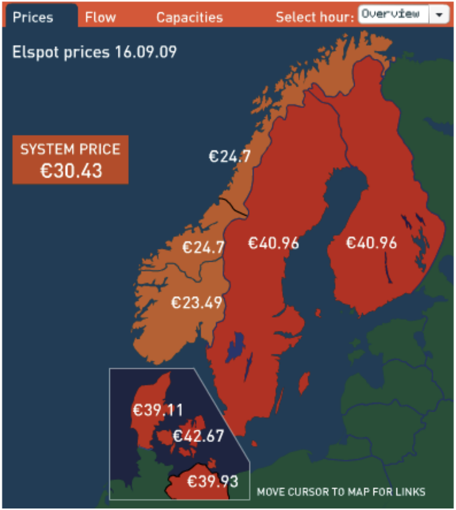
1. A survey of electricity and related markets
2. Stochastic analysis for independent increment processes
3. Stochastic models for energy spot price dynamics
4. Pricing of forwards and swaps based on the spot price
5. Applications to gas markets
6. Modelling forwards and swaps using the HJM approach
7. Constructing smooth forward curves in electricity markets
8. Modelling the electricity futures market
9. Pricing and hedging energy options
10. Analysis of temperature derivatives

Electricity markets

- a **flow commodity**
- non-storability issues
- physical and financial contracts
- focus on Nord Pool

Nordic Power Market

- Nord Pool was established in 1993.
- It covers Norway, Sweden, Finland and Denmark, and connects to Germany via the KONTEK bidding area.



Electricity markets

Nord Pool physical delivery contracts

Real-time (RT) markets

- These are operated by regional TSOs
- They include demand and supply-side bids giving prices and volumes, posted or changed close to operational time.
- A merit order is created for each hour and used to balance the system.
- There are also *ancillary services* markets.

Day-ahead (DA) markets

Elspot (Nordic market) closes at noon, at which point system prices are determined for each hour of the following day.

The Elbas market

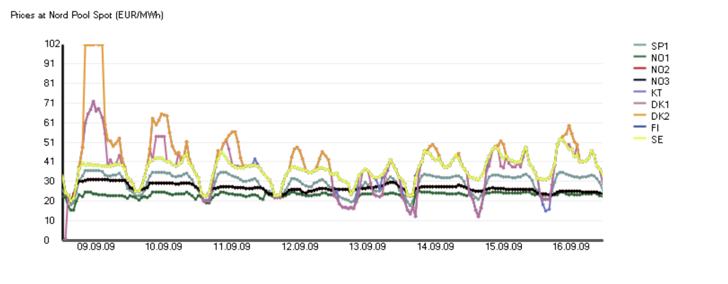
Elbas bridges the gap between Elspot and the RT market. It provides continuous power trading, opening when Elspot closes, and closing one hour before delivery.

Area prices

⊞ More information

Interval	Period	Currency	Area ☑ Mark all			[Right click to bookmark] [Print]
Hourly	Last 8 days	EUR	☑ System	☑ Norway 3	☑ Denmark East	Include graph in report ☑
			☑ Norway 1	☑ Kontek	☑ Finland	UPDATE REPORT
			☑ Norway 2	☑ Denmark West	☑ Sweden	

Report: Export data to Excel



Prices at Nord Pool Spot (EUR/MWh)

Electricity markets

Financial electricity contracts

- On Nord Pool, exchange traded contracts are based on a weighted average of the hourly system price over the delivery period.
- Electricity futures of this type are really *swap contracts*, committing to exchange the agreed futures price for the reference average system price (once it is known).
- On Nord Pool, there are four quarterly contracts introduced at the start of each year, for delivery commencing two years ahead of time, and a new yearly contract for three years ahead.
- There are also monthly contracts for six months ahead, weekly contracts for eight weeks ahead, and daily contracts for (up to) one week ahead, introduced each Thursday.
- There are also exchange-traded European-style options and contracts for differences.
- Other options are traded on an OTC basis.

Spot price modelling

- Common for financial assets: $S(t) = S(0)e^{X(t)}$, with $X(t) = \big(\mu t + \sigma B(t)\big)$.
- Log-returns, $X(t + \Delta t) - X(t)$, are independent and stationary, and **normal**.
- We can capture jumps and small time-scale leptokurtic behaviour by generalising to $S(t) = S(0)e^{L(t)}$, where $L(t)$ is a **Lévy process**.
- Cyclicality in price levels, jump sizes and frequence, etc., mean that models must be time-varying.
- Schwartz (1997) introduced $X(t)$ satisfying the OU process $dX(t) = \alpha\big(\mu - X(t)\big)dt + \sigma B(t)$.
- Generalising $B(t)$ to a Lévy process gives us an independent incremement (II) process.
- Seasonality can be incorporated by replacing $S(0)$ by a deterministic $\Lambda(t)$.
- They will also consider *additive models* - facilitating analytical pricing of swap contracts.

Spot price modelling

Spot power is not tradeable, although power futures are. The process(es) for the futures should be *semimartingales*, to eliminate arbitrage.
Connecting spot dynamics to futures prices leads to conditional expectation formulae for derivative contracts.

The closest thing to a spot price we can *observe* is the hourly (or half-hourly) price, although power futures are traded continuously.
Introduce a continuous spot process $\widetilde{S}(t)$ (unobserved), and hourly prices S_i^d (day d, hour i) (observed).
S_i^d is set before delivery commences, so we should have

$$S_i^d = \mathbb{E}\left[\int_{t_i^d}^{t_{i+1}^d} \widetilde{S}(u)du|\mathcal{F}_{t_i^d}\right].$$

Identifying the hourly price S_i^d as an observation of the spot at t_i^d is equivalent to approximating the integral by $\widetilde{S}(t_i^d)$ (time measured in hours).

Forward and swap pricing

Forward contract - for instantaneous delivery at a price $f(t,\tau)$.
Swap (futures) contract - for delivery between τ_1 and τ_2 at a price $F(t,\tau_1,\tau_2)$.
Spot - described by a stochastic process $S(t)$.

Forward pricing

The *payoff* at delivery time from a long forward position, entered at t, is

$$S(\tau) - f(t,\tau).$$

$f(t,\tau)$ is a fair price if $f(t,\tau) = \mathbb{E}_{\mathbb{Q}}\left[S(\tau)|\mathcal{F}_t\right]$, where $\mathbb{Q}$ is an equivalent martingale (risk-neutral) measure.

The complete market setting

If $S(t)$ is tradeable (perfectly liquid), f is uniquely determined:

$$f(t,\tau) = S(t)\mathrm{e}^{r(\tau-t)},$$

where r is the risk-free interest rate (assumed constant).

Forward and swap pricing

Storage costs and convenience yield

The hedging strategy used to derive the relation $f(t,\tau) = S(t)e^{r(\tau-t)}$ may come with costs (*storage costs*), but may also bring intangible benefits (*convenience yield*). A common approach is to model these as constant, proportional, cashflow streams, resulting in

$$f(t,\tau) = S(t)e^{(r+s-c)(\tau-t)}.$$

In this context, choosing an EMM, $\mathbb{Q}$, corresponds loosely to fixing the (net) convenience yield.

For power markets (and temperature markets), almost *any* $\mathbb{Q}$ will do, since the underying spot commodity is not storable. Forward price dynamics cannot be uniquely determined from arbitrage arguments.

Forward and swap pricing

According to the authors, the **rational expectations hypothesis** says that the foward price is the best available predictor of future spot prices:

$$f(t,\tau) = \mathbb{E}[S(\tau)|\mathcal{F}_t].$$

The theory of **normal backwardation** seeks to explain why this relation does not hold in practice: it posits the existence of a *risk premium*

$$RP(t,\tau) := f(t,\tau) - \mathbb{E}[S(\tau)|\mathcal{F}_t].$$

Many studies in various markets have found evidence of positive and negative risk premia.
Benth, Cartea and Kiesel (2006) discuss this in terms of the levels of risk aversion of buyers and sellers, and the market power of producers, relative to that of buyers.

Forward and swap pricing

A common practice, which the authors adopt, is to choose a parameterised class of measures $\mathbb{Q}$ and to fit the parameters to the observed risk premia.

Girsanov

A change in measure induces a change in the *drift*. If $B(t)$ is a $\mathbb{P}$-Brownian motion, then for any constant θ there is a measure $\mathbb{Q}$ equivalent to $\mathbb{P}$ such that $B^\theta(t) = B(t) - \theta t$ is a Brownian motion under $\mathbb{Q}$.

For example, if $S(t) = \mu t + \sigma B(t)$, then $RP(t, \tau) = \sigma\theta(\tau - t)$.

Esscher

The Esscher transform is a stucture-preserving change of measure that generalises the Girsanov transform to general II processes.
In this case, not only the drift terms will change, but also jump parameters, corresponding to *market prices of risk* associated with the various sources of risk.

Forward and swap pricing

Swap pricing

The payoff from a long position in an electricity futures (swap) contract is

$$\sum_{t_i=\tau_1}^{\tau_2} S(t_i) - (\tau_2 - \tau_1)F(t, \tau_1, \tau_2).$$

If settlement occurs at the end of the delivery period, we have

$$F(t, \tau_1, \tau_2) = \mathbb{E}_{\mathbb{Q}}\left[\frac{1}{\tau_2 - \tau_1}\sum_{t_i=\tau_1}^{\tau_2} S(t_i)|\mathcal{F}_t\right] = \mathbb{E}_{\mathbb{Q}}\left[\frac{1}{\tau_2 - \tau_1}\int_{\tau_1}^{\tau_2} \widetilde{S}(u)du|\mathcal{F}_t\right].$$

- The futures price can be viewed as an average of forward prices over the delivery period.
- If the model does not include a continuous spot price, then, for $t \in [t_i, t_{i+1})$, $\mathcal{F}_t = \mathcal{F}_{t_i}$, and $F(t, \tau_1, \tau_2) = F(t_i, \tau_1, \tau_2)$.

Forward and swap pricing

The Heath-Jarrow-Morton approach

- Model the forward rates (in interest rate markets) directly rather than the spot rates.
- This has been applied to forwards or to swaps in the the power context.
- With forwards, a problem is that prices are not directly observable.
  - Create a smooth forward curve by a fitting approach.
  - Deduce implied swap (futures) dynamics from the forward model.
- Another problem is the existence of overlapping contracts. The approach taken here is to include a minimal 'basis' set of contracts.
- Finally, one may rue the loss of a direct connection with the underlying spot price. This may not be a serious loss if one considers the spot to be a short-term futures contract (with delivery over the course of an hour).