Swing option pricing under jump-diffusion model

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Outline

- Introduction
- Snell Envelope
- Binomial Tree Method
- Monte Carlo Simulation
- Summary

Introduction

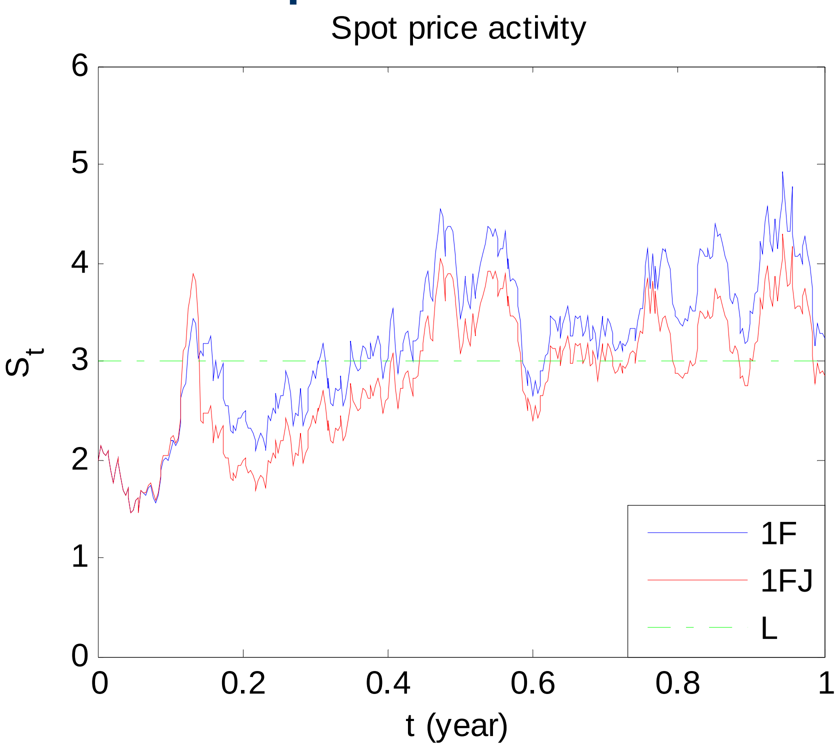
- Option Pricing Based on Stochastic Differential Equations
  - Step 1: Stochastic modeling
    - Choose the right stochastic differential equation model
    - Calibrate parameters
  - Step 2: Option pricing
    - Portfolio hedging: Partial differential equations
    - Fundamental Theorem of Asset Pricing: Calculating expectation of payoff functions

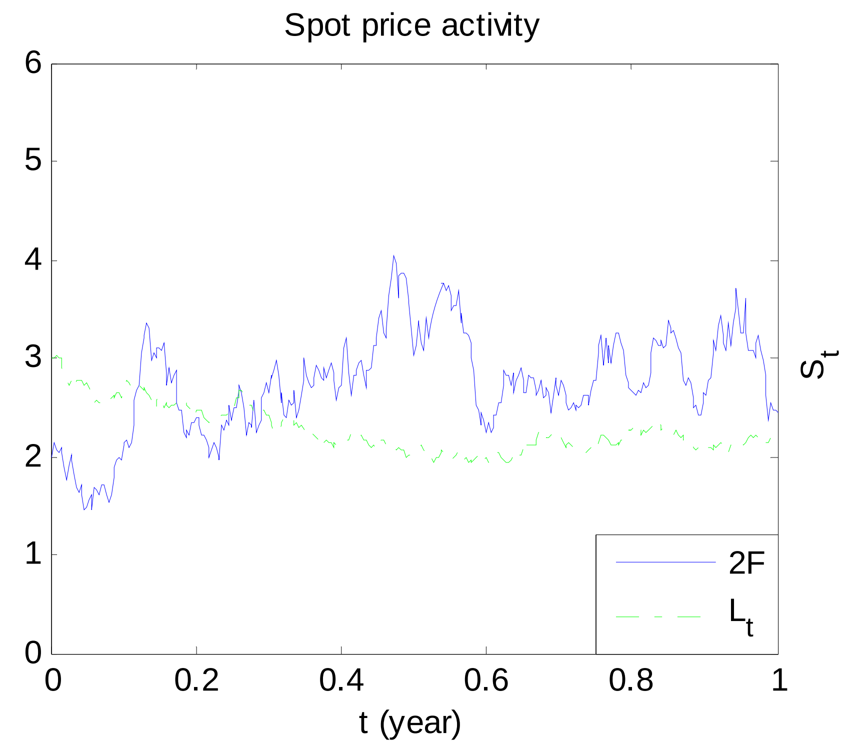
Introduction

- Mean-Reverting Diffusion Models
  - Widely used in energy markets
  - May contain only one stochastic process (one-factor) or two or more (multi-factor)
- Jump-Diffusion Models
  - Used to model markets with spikes
  - Option pricing may be difficult

Introduction

- Example





Introduction

- Swing Option
  - American Option
    - At any time t in [T_1, T_2], the option holder can exercise the right once to buy or sell an asset
    - Payoff for American call option at time t
      - $\mathrm{Max}[(S_t\text{-}K), 0]$, S_t the asset price, K the strike price
  - Swing Option
    - The option holder has m rights
    - Can exercise at any time $t_1<t_2<\ldots<t_m$ in [T_1, T_2] with a fixed volume Vol
    - Penalty if the total volume is not in [Vol_l, Vol_h]

Introduction

- Swing Option Pricing
  - Swing option is a good example among multi-exercise options
  - Jump models are closer to reality, but can make calculation more difficult
  - There is no analytical form solution

Snell Envelope

- Definition
  - For the process of payoffs $\{G_n\}_{0<n\le N}$ under spot prices $\{S_n\}$, with discount rate of each time interval r, the Snell envelope is defined as

$$X_N = G_N$$

$$X_n = \max(G_n, r\,\mathrm{E}[X_{n+1} \mid S_n]), \quad n = 1, \ldots, N-1$$

- Application on American Options
  - Exercise when G_t is equal to X_t
  - X_1 gives the expected payoff under optimal strategy

Snell Envelope

- Snell Envelope for Swing Options
  - Total rights: m
  - X_n^k: Snell envelope at time n with k remaining rights
  - $X_N^i = G_N \quad i = 1, 2, \ldots, m$

    $For\ 1 \le n < N$

    $X_n^1 = \max(G_n, r\,\mathrm{E}[X_{n+1}^1 \mid S_n])$

    $X_n^i = \max(G_n + r\,\mathrm{E}[X_{n+1}^{i-1} \mid S_n], r\,\mathrm{E}[X_{n+1}^i \mid S_n]), \quad i = 1, 2, \ldots, m$
  - X_1^m provides the optimal expected payoff

Snell Envelope

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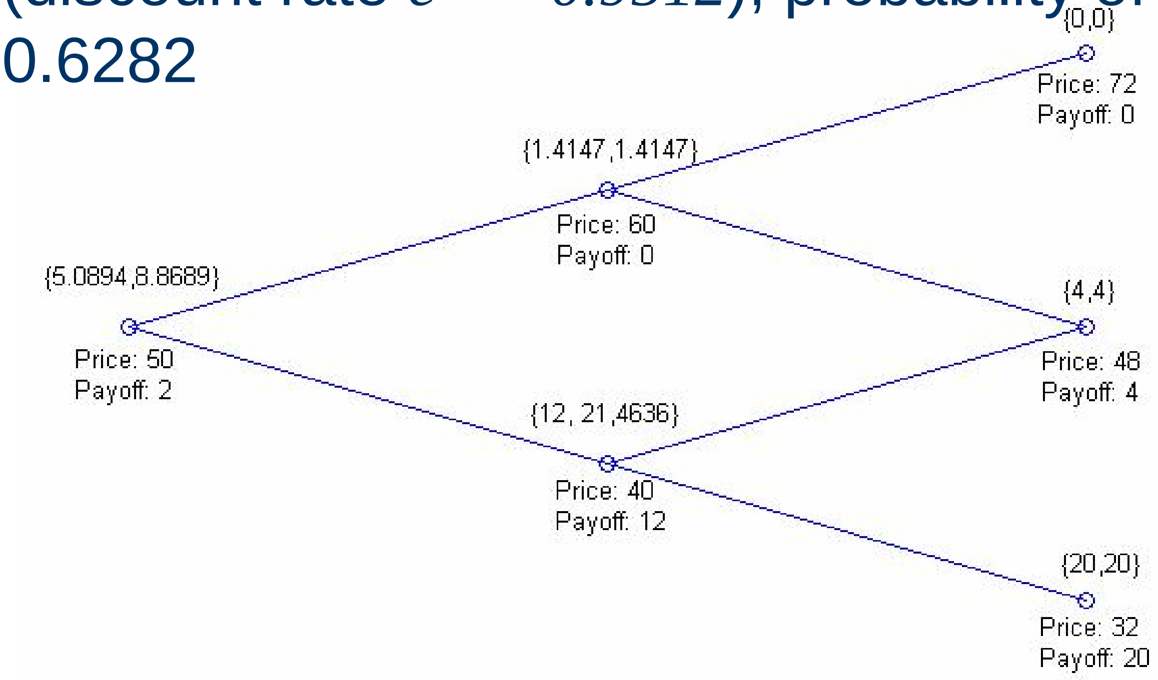
- Remark
  - Snell envelope can be applied to various kinds of multi-exercise options
  - Optimal stopping strategy can be achieved
  - There are several numerical ways to implement the method

Binomial Tree Method

- Concept (K. Amin, T. Ware et al.)
  - Use backward programming to achieve all the values of $\mathrm{E}[X_{n+1}^{i} \mid S_n]$
- Construction
  - No jump models: Binomial Tree
  - Models with jumps: Multinomial Tree
- Remark
  - Can be extended to forests on multi-factor model case

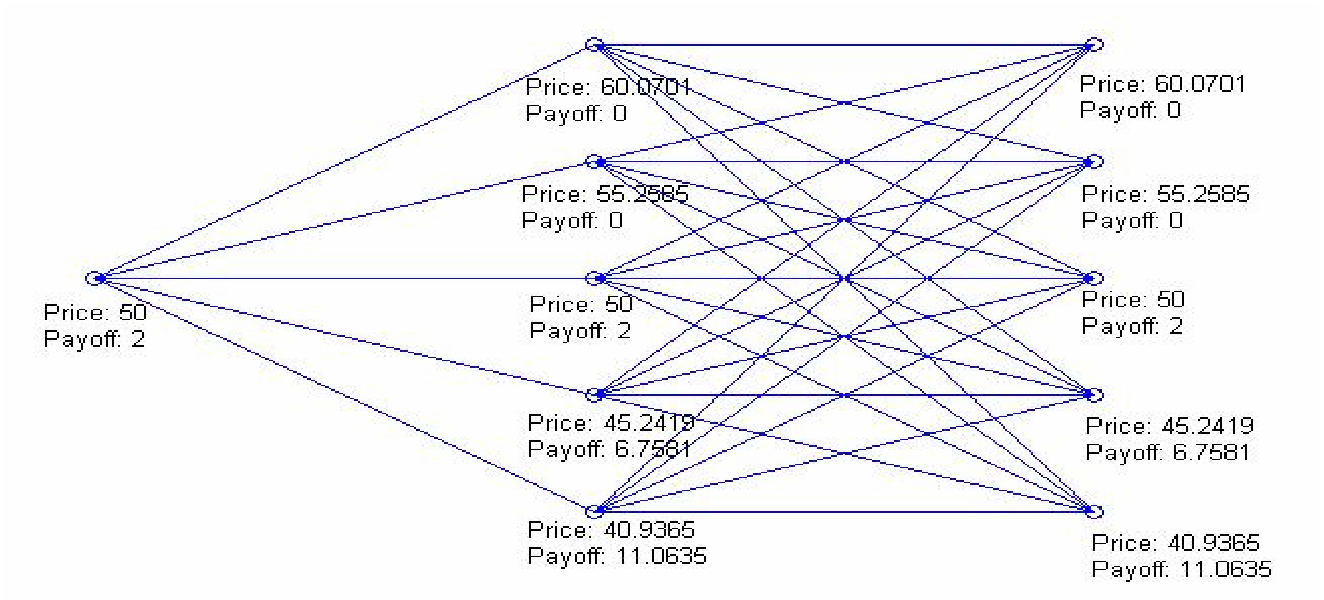
Binomial Tree Method

- Example (Binomial Tree)
  - Put swing option, strike price $52, interest rate 5%(discount rate $e^{-5\%}=0.9512$), probability of going up 0.6282



Binomial Tree Method

- Example (Multinomial Tree)
  - Put swing option, strike price $*52*



Monte Carlo Simulation

- Concept (E. Schwartz, N. Meinshausen et al.)
  - Use simulations and backward regression to achieve $\mathrm{E}[X_{n+1}^{i} \mid S_n]$
- Remark
  - The estimate is biased
  - The method can be applied to various kinds of stochastic models

Monte Carlo Simulation

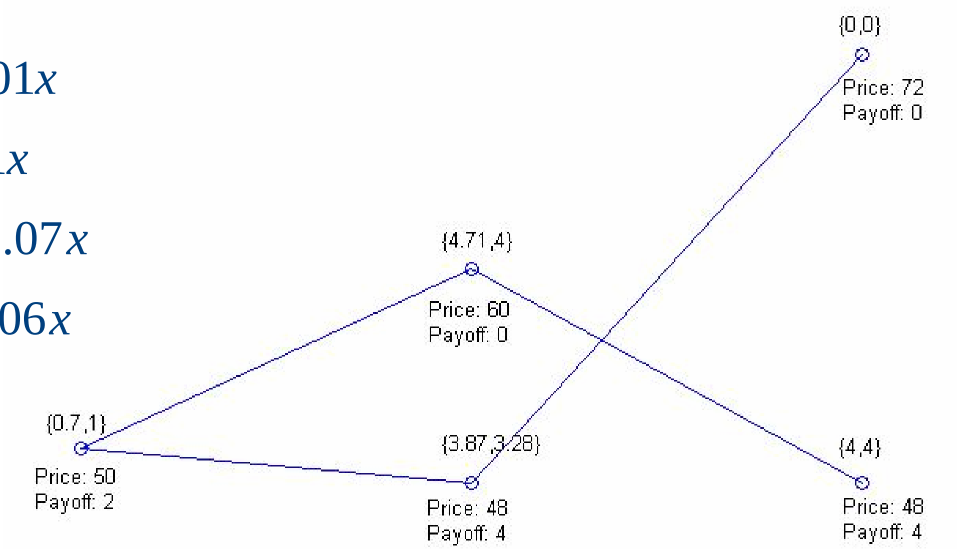
- Example
  - Put swing option, strike price $52, discount rate 0.9512

$$E[X_1^1 \mid S_1 = x] = 0.2 + 0.01x$$

$$E[X_1^2 \mid S_1 = x] = -4 + 0.1x$$

$$E[X_2^1 \mid S_2 = x] = 0.51 + 0.07x$$

$$E[X_2^2 \mid S_2 = x] = 0.4 + 0.06x$$



Summary

- Snell Envelope
  - Can be applied to price many kinds of options
- Binomial/multinomial Tree Method
  - Straight and clear for one-factor models
  - Converge to the accurate result
  - Can be extended to implement multi-factor models
- Monte Carlo Method
  - Biased estimate
  - Applicable on any stochastic model

Thank you!

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