The Analytical Finance Package

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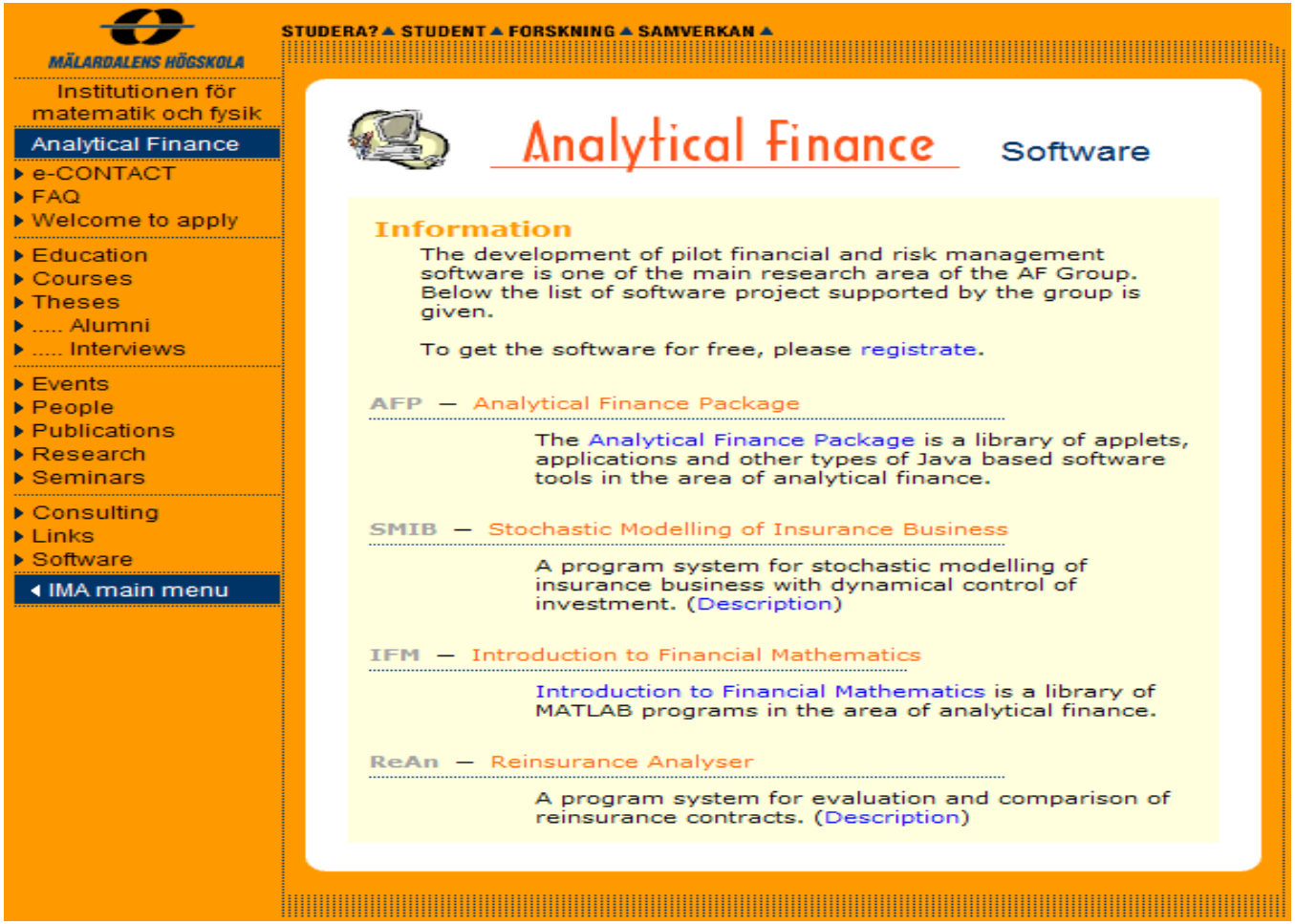
Introduction

Analytical Finance group at the Mälardalen University develops the Analytical Finance Package (AFP). This is the library of Java applets in the area of analytical finance. The project was initiated in 2003, and it is realising on a permanent base.
Professor Dmitrii Silvestrov and Dr. Anatoliy Malyarenko are leaders of the project. They develop AFP in collaboration with other members of Analytical Finance group and students of the Bachelor programme Analytical Finance and the Master programme Financial Engineering realising at the Mälardalen University.
The initial list of subject areas and number of items available now in each subject area are the following: A — Simulation of pricing processes (11 items); B — Estimation of pricing processes (0 items); C — Evaluation of financial contracts (32 items). The AFP is free software. The access to the program library AFP is available from the web-address:

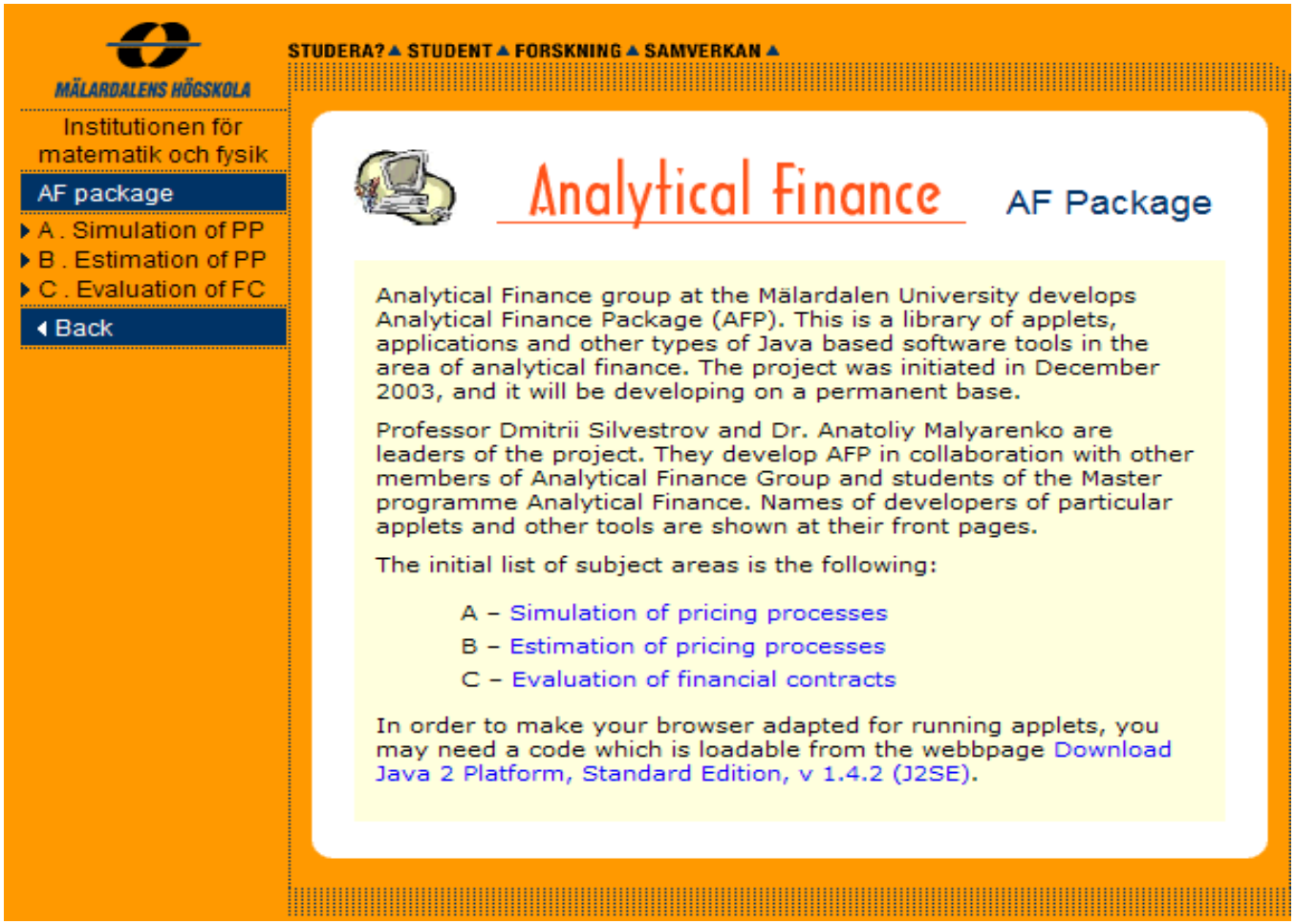
http://www.mdh.se/ima/analyticalfinance/af1.software.shtml

The Analytical Finance Web page

The Software Web page



The Analytical Finance Package Web page



Dmitrii Silvestrov, Anatoliy Malyarenko

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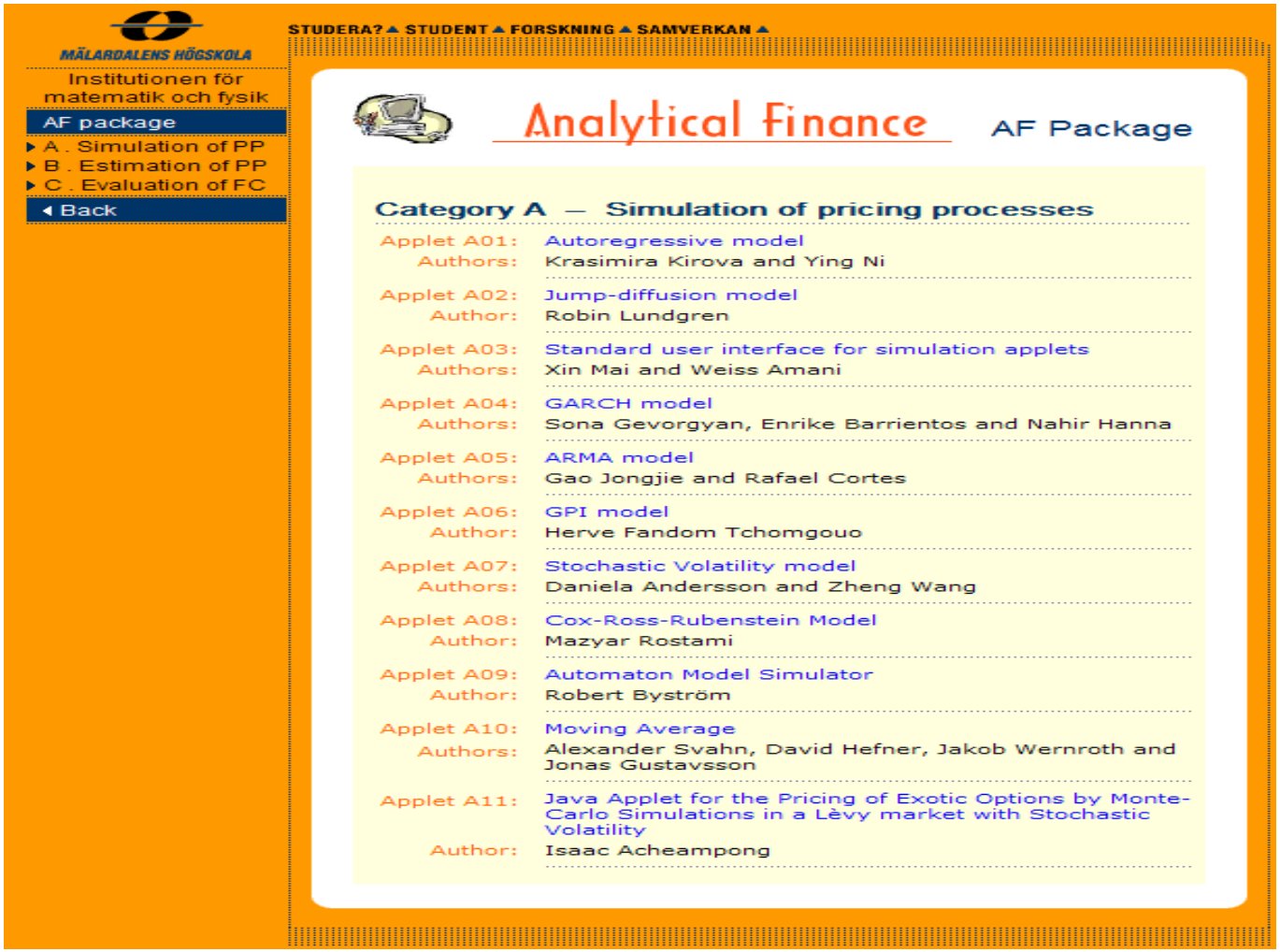
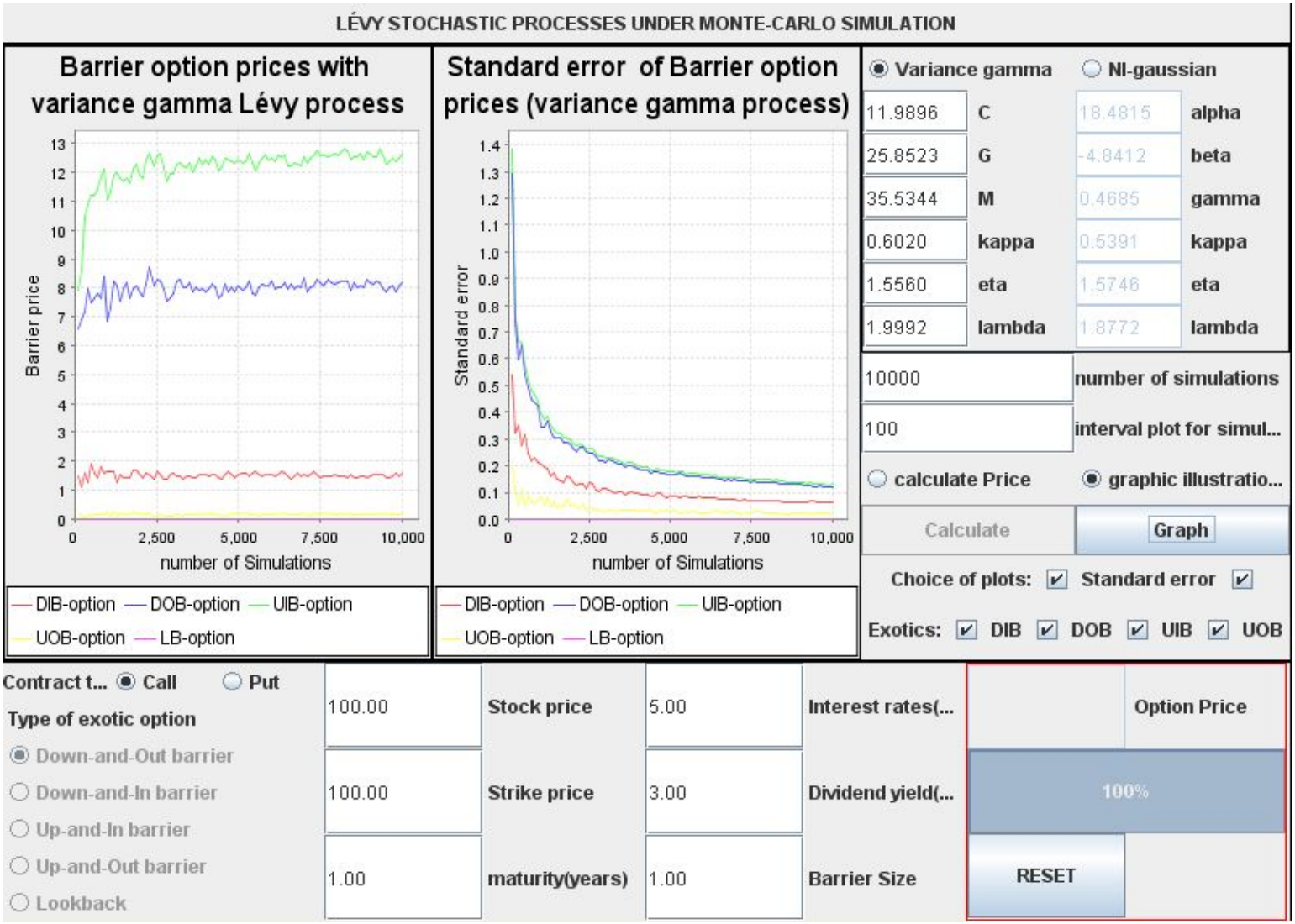
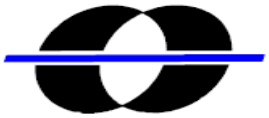
Category A — Simulation of pricing processes



Applet A11 — Pricing of Exotic Options in a Lèvy market with Stochastic Volatility (Isaac Acheampong)



The Master thesis



MÄLARDALENS HÖGSKOLA
Institutionen för Matematik och Fysik

Code: MdH.IMa.Mat.0061-(2006)10p-AF

MASTER THESIS IN MATHEMATICS /APPLIED MATHEMATICS

Java Applet for the Pricing of Exotic Options by Monte-Carlo Simulations in a Lèvy market with Stochastic Volatility

by

Isaac Acheampong

Notation

Table: Notation

Symbol	Explanation
t	Current time
$S(t)$	The stock price at time t
$\sigma^2(t)$	The volatility of the underlying stock at time t
r	Risk-free interest rate
α	The speed of volatility mean reversion
θ	The long run mean of the volatility process
β	The volatility of the volatility process
ρ	The correlation coefficient
$B^1(t), B^2(t)$	Two independent Brownian motions
K	The strike price of an Asian call option
T	The expiration time of the option contract

The formulation of a problem

The price process and the volatility process follow the *Heston model* described by the following system of stochastic differential equations in Itô form.

$$dS(t) = rS(t)\,dt + S(t)\sigma(t)\,dB^1(t),$$
$$d\sigma^2(t) = \alpha(\theta - \sigma^2(t))\,dt + \beta\,\sigma(t)(\rho\,dB^1(t) + \sqrt{1-\rho^2}\,dB^2(t)),$$

with $S(0) = S_0$ and $\sigma^2(0) = \sigma_0^2$. We would like to calculate the price of an Asian call option on this asset.

Solution, step 1

Introduce the process

$$I(t) = \int_0^t S(s)\,ds,$$

and rewrite the Heston model in the Itô integral form

$$\begin{aligned}
S(t) &= S_0 + \int_0^t rS(s)\,ds + \int_0^t S(s)\sigma(s)\,dB^1(s),\\
\sigma^2(t) &= \sigma_0^2 + \int_0^t \alpha(\theta - \sigma^2(s))\,ds + \int_0^t \beta\sigma(s)(\rho\,dB^1(s)\\
&\quad + \sqrt{1-\rho^2}\,dB^2(s)),\\
I(t) &= 0 + \int_0^t S(s)\,ds.
\end{aligned}$$

Let $f(S(T), \sigma^2(T), I(T)) = \max\{I(T)/T - K, 0\}$. Then the price of the Asian call option is $C = \mathrm{E} f(S(T), \sigma^2(T), I(T))$.

Solution, step 2

Introduce the following notation.

$$B^0(s) = s,$$
$$\mathbf{Y}(t) = (S(t), \sigma^2(t), l(t))',$$

and rewrite the model in the Stratonovich integral form

$$\mathbf{Y}(t) = \mathbf{Y}(0) + \sum_{i=0}^{2} \int_0^t \mathbf{V}_i(\mathbf{Y}(s)) \circ dB^i(s), \qquad (1)$$

where $\mathbf{V}_0$, $\mathbf{V}_1$, $\mathbf{V}_2 : \mathbb{R}^3 \mapsto \mathbb{R}^3$ are the following functions.

$$\mathbf{V}_0((y_1, y_2, y_3)') = (y_1(r - y_2/2 - \rho\beta/4), \alpha(\theta - y_2) - \beta^2/4, y_1)',$$
$$\mathbf{V}_1((y_1, y_2, y_3)') = (y_1\sqrt{y_2}, \rho\beta\sqrt{y_2}, 0)',$$
$$\mathbf{V}_2((y_1, y_2, y_3)') = (0, \beta\sqrt{(1-\rho^2)y_2}, 0)'.$$

Solution, step 3

Let ξ_1, ξ_2, ξ_3, and ξ_4 be four independent standard normal random variables. Introduce the following functions.

$$\mathbf{W}_{1,s} = -\frac{1}{2}s\mathbf{V}_0 + \frac{\sqrt{3}}{2}\xi_1\sqrt{s}\mathbf{V}_1 + \frac{\sqrt{3}}{2}\xi_3\sqrt{s}\mathbf{V}_2,$$

$$\mathbf{W}_{2,s} = \frac{3}{2}s\mathbf{V}_0 - \frac{5\sqrt{3}}{6}\xi_1\sqrt{s}\mathbf{V}_1 + \frac{\sqrt{6}}{3}\xi_2\sqrt{s}\mathbf{V}_1 - \frac{5\sqrt{3}}{6}\xi_3\sqrt{s}\mathbf{V}_2 + \frac{\sqrt{6}}{3}\xi_4\sqrt{s}\mathbf{V}_2.$$

Solution, step 4

Consider the following two systems of ordinary differential equations (ODE):

$$\frac{d\mathbf{y}(t)}{dt} = \mathbf{W}_{j,s}(\mathbf{y}(t)), \quad \mathbf{y}(0) = \mathbf{y}_0, \qquad j = 1,2, \quad s > 0.$$

These systems are the ODE-valued random variables in the sense of Ninomiya and Ninomiya (2008).
Let a_{ik} be an $M \times M$ matrix with $a_{ik} = 0$ for $i \leq k$, and let b_i be a vector of length M (specially designed). The *M-stage Runge–Kutta method of order m* for the above mentioned systems is written as follows.

$$\mathbf{y}^{(i)} = \mathbf{y}_0 + s \sum_{k=1}^{M} a_{ik} \mathbf{W}_{j,s}(\mathbf{y}^{(k)}), \quad 1 \leq i \leq M,$$

$$g(\mathbf{W}_{j,s})(\mathbf{y}_0) = \mathbf{y}_0 + s \sum_{i=1}^{M} b_i \mathbf{W}_{j,s}(\mathbf{y}^{(i)}).$$

Solution, the last step

Let n be a positive integer. Define recursively the second order scheme as the following sequence of random vectors:

$$\mathbf{Y}_0^{(n)} = \mathbf{Y}(0),$$
$$\mathbf{Y}_{(k+1)T/n}^{(n)} = g(\mathbf{W}_{1,1}) \circ g(\mathbf{W}_{2,1})(\mathbf{Y}_{kT/n}^{(n)}), \qquad 0 \le k \le n-1,$$

and define the third order scheme by the Romberg extrapolation:

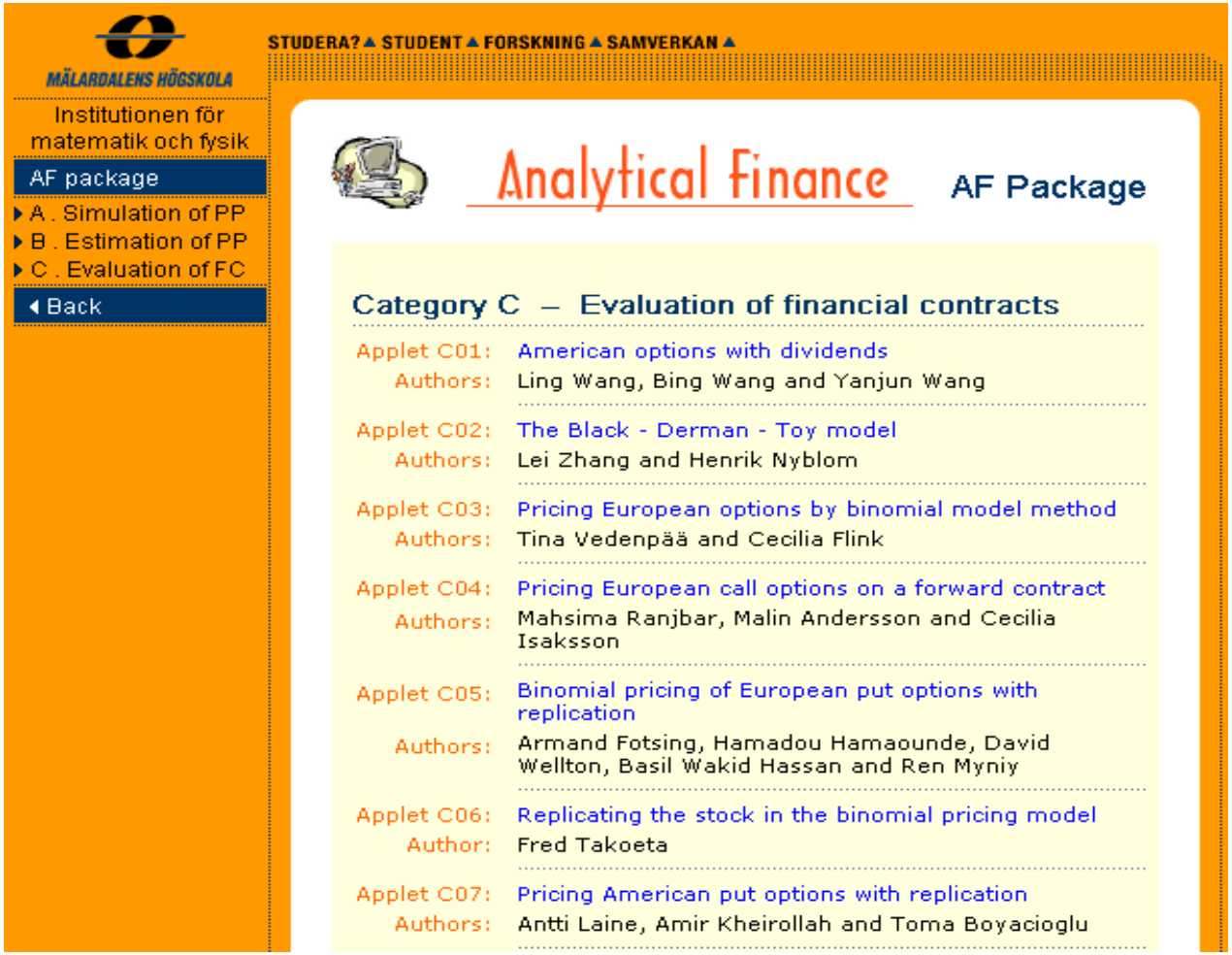
$$\mathbf{X}_T^{(2n)} = \frac{4}{3}\mathbf{Y}_T^{(2n)} - \frac{1}{3}\mathbf{Y}_T^{(n)}.$$

Ninomiya and Ninomiya (2008) proved that the average of $\mathbf{X}_T^{(2n)}$ approximates system (1). In particular,

$$\lim_{n\to\infty} \mathrm{E} f(\mathbf{X}_T^{(2n)}) = \mathrm{E} f(\mathbf{Y}(T)).$$

To calculate $\mathrm{E} f(\mathbf{X}_T^{(2n)})$, use quasi Monte Carlo.

Category C — Evaluation of financial contracts



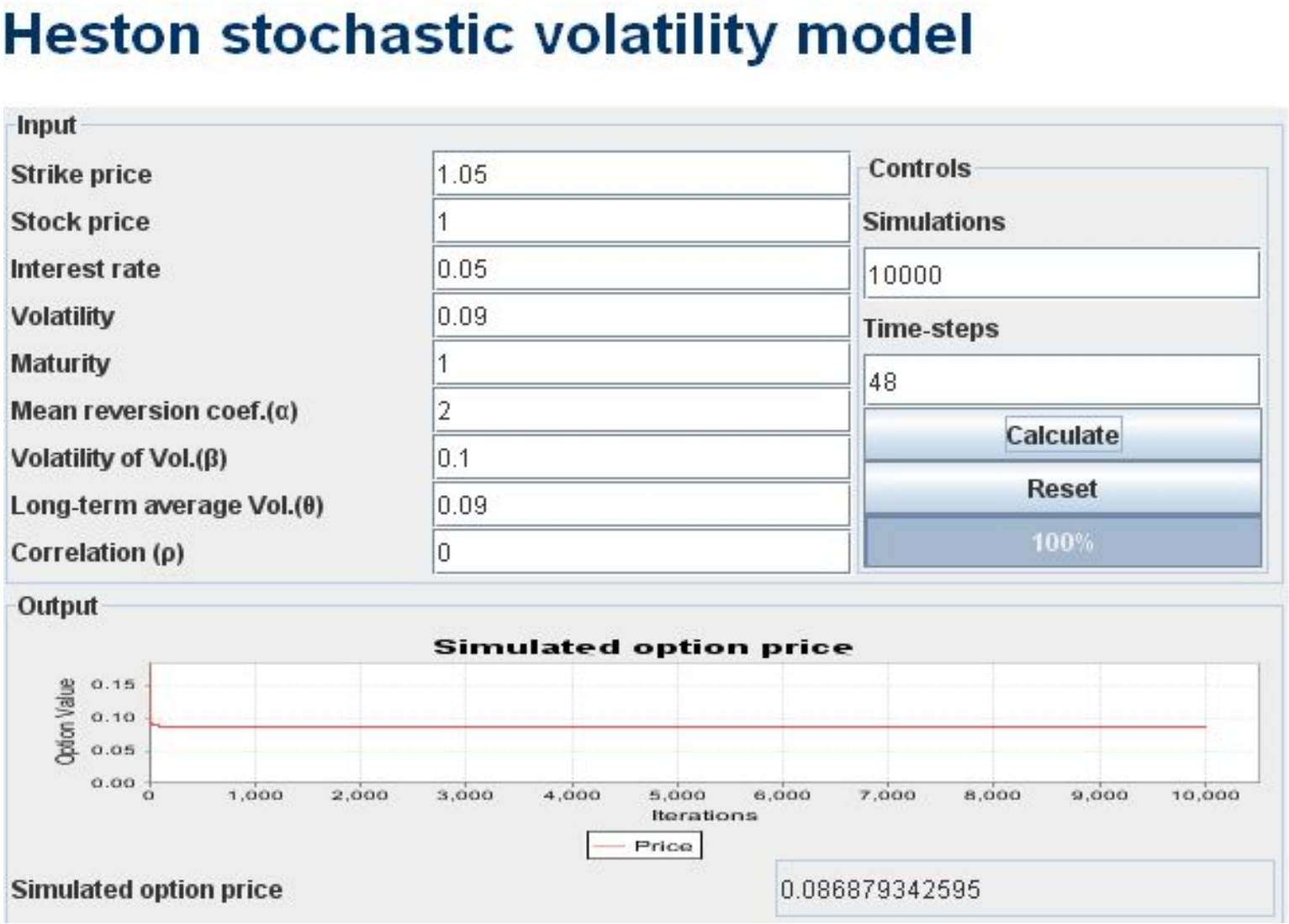
Category C — Evaluation of financial contracts (continue)

Applet C08:	The Hull-White model
Author:	Aminur Roshid
Applet C09:	Bond Price Calculator
Authors:	Aminur Roshid, Isaac Acheampong, Peter Agyemang-Mintah and Yue Song
Applet C10:	Simulation of the short interest rate in the Vasicek model
Authors:	Natalia Spas'ka and Olexander Sheychenko
Applet C11:	Eurobonds problem
Authors:	Kamila Giedrojc, Ewa Tropak and Romans Obrezkovs
Applet C12:	Investor's problem
Authors:	Elizabeta Tudzarovska, Izabela Matusiak, Kwok-wai Choy, Sophia Abdi Hassan and Khadija Khapasi
Applet C13:	Bond reselling problem
Authors:	Malgorzata Andros, Matti Simperi and Piotr Liszewski
Applet C14:	Loan repayment applet
Authors:	Osei Benjamin Kwesi Amoako, George Manteaw Anobah, Aleksandra Sroka and Jacek Zniszczol
Applet C15:	Insurance problem
Authors:	Beata Lubecka, Peter Malosha Mayunga, Maziar Saei Aghmiouni and Marek Geringer De Oedenberg

Category C — Evaluation of financial contracts (continue)

Applet C16:	Pricing put options using explicit finite difference method in Java graphical user interface
Author:	Yue Song
Applet C17:	A Java program for pricing options using the trinomial tree
Authors:	Youmbi Etien Kalame
Applet C18:	Pricing futures using the two-period binomial model in Java
Authors:	Minyi Ren and Wakid Hassan Basil
Applet C19:	Pricing convertible bonds with Monte Carlo simulations
Author:	Cecilia Isaksson
Applet C20:	A Java applet for credit risk estimation with Wishart multivariate stochastic volatility
Author:	Amoako Osei Benjamin Kwesi
Applet C21:	A Java applet for pricing convertible bonds with credit risk
Author:	Ntui Charles Etang
Applet C22:	A Java applet for simulation of economy with borrowers under costly defaults
Author:	Basil Wakid Hassan

Applet C34 — Pricing Asian options under Heston's model using the new Ninomiya weak approximation scheme (Boyko Vasilev)



The Bachelor thesis

MÄLARDALEN UNIVERSITY
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School of Education, Culture and Communication
Division of Applied Mathematics

Code: MDH.UKK.TM.0000 (15hp) 2008 AF

BACHELOR THESIS IN MATHEMATICS/APPLIED MATHEMATICS

The Java applet for pricing Asian options under Heston's model using the new Ninomiya weak approximation scheme and quasi-Monte Carlo

by

Boyko Vasilev

For further reading

Ninomiya, M. and Ninomiya, S. (2008),
A new weak approximation scheme of stochastic differential equations by using the Runge–Kutta method,
arXiv:0709.2434v2 [math.PR] 5Jan 2008.

Greetings from Sydney!