A two factor and Levy based approach for Gas futures and spot modelling

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Outline

About Natural Gas

- Natural gas is one of the cleanest, cheapest and most efficient sources of energy.
- Alberta is home to a large natural gas resource base and accounts for just over 80 per cent of the natural gas produced in Canada.
- 87 trillion cubic feet (Tcf) of recoverable, conventional natural gas is still beneath our feet.
- Flammable gas, commonly used to fuel household appliances, Heating homes and businesses

Gas in Alberta

- 75 per cent of the natural gas consumed in Alberta is used by the industrial sector (including electricity generation).
- Displays seasonality, need models to capture it.
- Natural gas futures and options on futures are sold in the market.
- Natural gas storages used as a hedging instrument.

OU process

The commonly used process to model natural gas behaviour is the mean reverting Ornstein-Uhlenbeck (OU) process.
This is the most popular one factor model in natural gas spot simulation.
The OU process is defined by

$$dS_t = \theta(\mu - S_t)dt + \sigma dW_t$$

where θ is the speed of mean reversion, μ is the value that the spot price reverts to, σ is the diffusion term and W_t is a Weiner process.

OU process

In this setting, the futures price is given by

$$F(t,T) = E_Q[S_T exp(\int_t^T (r_s)ds)|F_t]$$

For simplicity, we assume a constant risk free interest rate.

$$E(S_t) = S_0 e^{-\theta t} + \mu(1 - e^{-\theta t})$$

$$\begin{aligned} Var(S_t) &= \frac{\sigma^2}{2\theta} \\ Cov(S_s, S_t) &= E[(S_s - E[S_s])(S_t - E[S_t])] \\ &= \frac{\sigma^2}{2\theta\, e^{-\theta(s+t)}(e^{2\theta(s\wedge t)} - 1)} \end{aligned}$$

Gas models

- In commodities, there is also a convenience yield factor.
- Convenience of not having to store gas, is embedded in a futures price.
- Futures equation is not sufficient to capture dynamics of the real futures curve.
- Gibson-Schwartz introduced convenience yield models.

Convenience Yield

- A factor implied by the futures or other derivative prices of commodities.
- Net benefit minus the cost of holding energy.
- Unlike financial derivatives, storage of energy products is costly.
- Physical ownership of commodity, carries an associated flow of services.
- Agent has the option of flexibility with regards to consumption, but this decision of postpone consumption implies storage expenses.

Convenience Yield

- δ_t= benefit of direct access - cost of carry.
- $$F(t, T) = S_t E_Q[exp(\int_t^T (r_s - \delta_s)ds]$$

Gibson-Schwartz Model

Let (Ω, F, P) be a probability space under a filtration $\{F_t\}_{t \geq 0}$. According to the Gibson-Schwartz model, under the risk-neutral measure Q,

$$dS_t = (r_t - \delta_t)S_t dt + \sigma S_t dW_t^1,$$
$$d\delta_t = \kappa(\theta - \delta_t)dt + \gamma dW_t^2,$$

where W_1 and W_2 are correlated Weiner processes with $dW_1 dW_2 = \rho dt$.

Gibson-Schwartz Futures Price

$$F(t,T,S_t) = S_t e^{\int_t^T r_s ds} e^{B(t,T)\delta_t + A(t,T)}$$

where

$$\begin{aligned}
B(t,T) &= \frac{e^{\kappa T} - 1}{\kappa}, \\
A(t,T) &= \frac{\kappa\theta + \rho\sigma\gamma}{\kappa^2}(1 - e^{-\kappa(T-t)} - \kappa(T-t), \\
&+ \frac{\gamma^2}{\kappa^3}(2\kappa(T-t) - 3 + 4e^{-\kappa(T-t)} - e^{-2\kappa(T-t)}).
\end{aligned}$$

Gibson-Schwartz model

- Schwartz used an extra stochastic interest rate.
- Did not yield qualitative improvements, so it is not used in literature.

Pilipovic Model

- Pilipovic's idea was to have a model that is more intuitive in terms of the second factor.
- Consider the long run mean L_t, as the equilibrium price.
- That is, the price of the commodity when the supply and the demand are in balance, and S_t as the spot price, both at time t, then the difference between the two represents the measure of the market imbalance.
- $\delta_t = S_t - L_t$, where δ_t is the convenience yield

Pilipovic Model

$$d\tilde{S}_t = \alpha(L_t - S_t)dt + \sigma S_t d\tilde{W}_t^1$$
$$dL_t = \mu L_t dt + \gamma L_t dW_t^2,$$

She says that the futures curve should be modelled by first stripping off the seasonality, i.e., $F(t,T) = F^{UND}(t,T) +$ seasonality contribution. The spot process being of an exponential affine form, also has an explicit form for its futures price.

$$F(t,T) = (S_t - L_t)e^{-(\alpha+\lambda\gamma)(T-t)} + L_t e^{(\mu-\lambda\gamma)(T-t)}$$

Xu's Model

- Xu calibrated a generalization of this model, that we worked with as well.

$$\begin{aligned} S_t &= f(t) + X_t \\ dX_t &= \alpha(L_t - X_t)dt + \sigma X_t dW_t^1 \\ dL_t &= \mu(\gamma - L_t)dt + \tau L_t dW_t^2 \\ f(t) = bt &+ \sum_{j=1}^{2} \beta_j cos2\pi jt + \eta_j sin2\pi jt. \end{aligned}$$

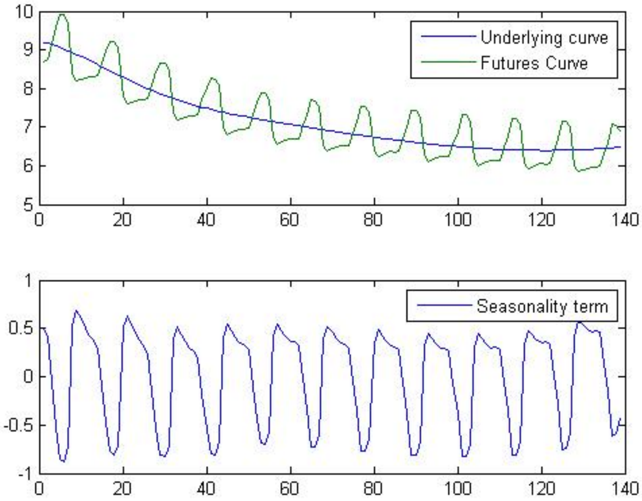
- The seasonality term was constant, but allows us to work with more data.
- Illustration of the problem - Simulations of the Gas futures curve.

Exponential seasonality.

The following is based on Pilipovic:
If $F(t,T) = f(T-t) + X(t,T)$ is our futures equation.

$$\begin{aligned} X(t,T) &= \beta_1 exp(-\gamma_1(rfc((T-t)-t_1^C))^2) + \beta_2 exp(-\gamma_2(rfc((T-t) \\ &+\beta_3 exp(-\gamma_3(rfc((T-t)-t_3^C))^2)); \end{aligned}$$

Stripping off the seasonality.



- Underlying seasonality is not satisfactory.
- Parameters estimated by optimization (used matlab function: fminsearch)
- Estimated parameters: $\beta_1 = 3.9031$, $\gamma_1 = 0.0099$, $t_1^C =$ -5.1825
- $\beta_2 = 3.8450$, $\gamma_2 =$ -0.0025, $t_2^C =$ -5.1756
- $\beta_3 = 1.5332$, $\gamma_3 = 0.1846$, $t_3^C =$ -5.4838

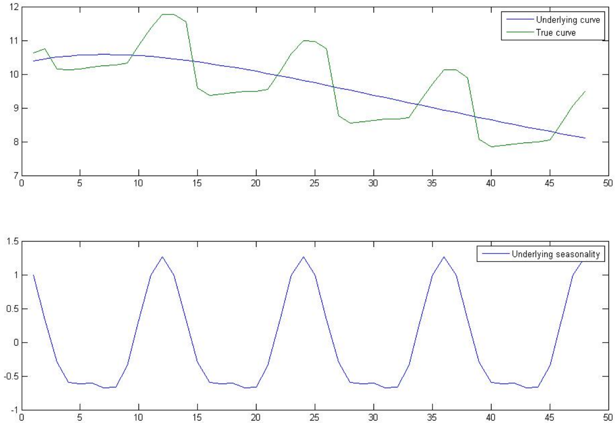
Nielsen-Siegel type futures curve

$$X(\tau_i) = l + s(\frac{1 - e^{-\lambda\tau_i}}{\lambda\tau_i}) + c(\frac{1 - e^{-\lambda\tau_i}}{\lambda\tau_i} - e^{-\lambda\tau_i}) + v(\tau_i) \quad (1)$$

where $\tau_i = T - t$, and l, s, c, v are parameters.
Now Nelson-Siegel suggests to use $\lambda = 0.0609$, so that the equation can now be estimated via regression. So we use this approach and set the forward curve to equal

$$F(t, T_i) = \sum_{i=1}^{2}[u_i sin(2rfc\pi(T-t)+v_i cos(2\pi rfc(T-t)]+X(T_i-t) \quad (2)$$

Stripping off the seasonality



Nielsen-Siegel type futures curve

- The seasonality obtained is more consistent with what is seen.
- Regression model - ease and speed.
- The residual error is simply bootstrapped.
- Following is a simulation with seasonalities bootstrapped at the beginning of the new contract every month.

Xu's Model adapted

- In order to model the underlying curve, we use Xu's equation without the seasonality

$$dS_t = \tilde{\alpha}(L_t - S_t)dt + \sigma S_t^r dW_t^1$$
$$dL_t = \tilde{\mu}(\tilde{\gamma} - L_t)dt + \tau L_t^r dW_t^2$$

- Rather than simply using the regression coefficients to resimulate the curve, we want to find an economic intuition behind the curves dynamics.
- α, μ tells us the rate of mean reversion of the Spot price, "long run mean".
- γ tells us where the "long run mean" reverts to.

Futures model

Usefulness of the above futures equation is its “affineness”, has explicit form of characteristic function, through which we have the futures price.

If we have “m” contracts whose data we have available ahead of the observation time t, the expiry time for these contracts is $T_t i$

So the theoretical futures price is given by,

$F'^{\tilde{\theta}}(t, T_{ti}, S_t, L_t) = A_i + B_i L_t$ where

$A_i = e^{\tilde{\alpha}(t-T_{ti})} S_t + \frac{\tilde{\mu}\tilde{\gamma}}{\tilde{\alpha}-\tilde{\mu}}(e^{\tilde{\alpha}(t-T_{ti})} - 1) - \frac{\tilde{\alpha}\tilde{\gamma}}{\tilde{\alpha}-\tilde{\gamma}}(e^{\tilde{\mu}(t-T_{ti})} - 1)$ and

$B_i = \frac{\tilde{\alpha}}{\tilde{\alpha}-\tilde{\mu}}(e^{\tilde{\mu}(t-T_{ti})} - e^{\tilde{\mu}(t-T_{ti})})$

Finding the hidden term L_t

- The set of parameters that we have are $\tilde{\theta} = [\tilde{\alpha}, \tilde{\mu}, \tilde{\gamma}, \tau, \sigma]$
- From theoretical futures price equation, the parameters only depend on $\tilde{\alpha}, \tilde{\mu}, \tilde{\gamma}$.
-

$$L_t(\tilde{\theta}) = argmin_{L_t} \sum_{i=1}^{m} (F'^{\tilde{\theta}}(t, T_{ti}, S_t, L_t) - F'(t, T_{ti}))^2 \quad (3)$$

- Substituting our equation $F'^{\tilde{\theta}}(t, T_{ti}, S_t, L_t)$ in the above, and realizing that we are simply calculating the distance in euclidian space, $\mathbb{R}^m$.

Finding the hidden term L_t

- We obtain

$$\begin{aligned} L_t(\tilde{\theta}) &= argmin_{L_t}||A + BL_t - F'(t, T_t)||_2 \\ &= argmin_{L_t}||A - F'(t, T_t) + BL_t||_2 \\ &= \frac{||BF'(t, T_t) - AB||_1}{||B||_2} \end{aligned}$$

- This is the same as,

$$L_t(\tilde{\theta}) = \frac{\sum_{i=1}^{m}(B_i F'(t, T_{ti}) - A_i B_i)}{\sum_{i=1}^{m} B_i^2} \qquad (4)$$

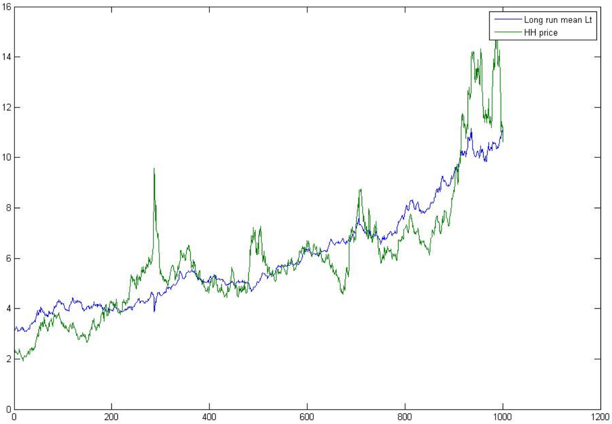
- Now to find out parameters $\tilde{\theta}$:

$$\tilde{\theta} = argmin_{\tilde{\theta}} \sum_{t=1}^{n} \sum_{i=1}^{m} (F'^{\tilde{\theta}}(t, T_{ti}, S_t, L_t) - F'(t, T_{ti}))^2$$

Finding the hidden term L_t

- Parameters: $\tilde{\alpha} = 0.005885$, $\tilde{\mu} = 3.2497$, $\tilde{\gamma} = 0.000637$
- These are our risk neutral parameters.
- We use the risk neutral parameters to find L_t, from the equation.

L_t revealed


16
14
12
10
8
6
4
2
0
0
200
400
600
800
1000
1200
Long run mean Lt
HH price

Model Calibration

The two factor model can be estimated via the simulation equation, that is given by

$$S_{t+1} - S_t = \alpha(L_t - S_t) + \sigma S_t^r (W_{t+1}^1 - W_t^1)$$
$$L_{t+1} - L_t = \mu(\gamma - L_t) + \tau L_t^r (W_{t+1}^2 - W_t^2)$$

Now $W_{t+1}^i - W_t^i \sim N(0,1)$, so the probability densities for each S_t and L_t, can easily be calculated.
From the simulation equation, we have

$$f(\theta, S_{t+1}[S_t, L_t]) \sim N(S_{t+1} - S_t - \alpha(L_t - S_t), \sigma S_t^r)$$

and

$$f(\theta, L_{t+1}|L_t) \sim N(L_{t+1} - L_t - \mu(\gamma - L_t), \tau L_t^r)$$

Model Calibration

If θ are our parameters then,

$$f(\theta, [S_{t+1}, L_{t+1}] | [S_t, L_t]) = f(\theta, S_{t+1}[S_t, L_t]) f(\theta, L_{t+1} | L_t)$$

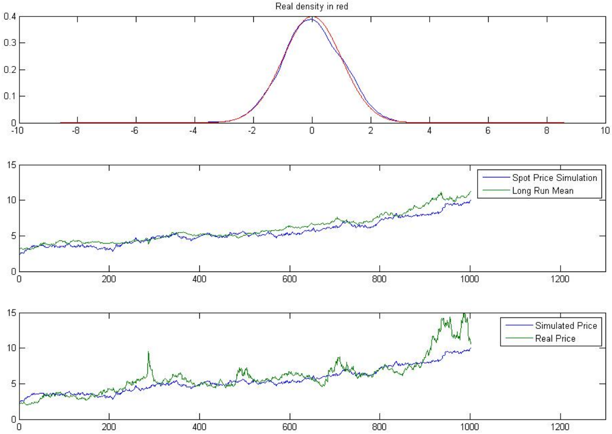
So since our conditional density function is known, we can easily use maximum likelihood to estimate the parameters.
Xu found the explicit formulas for these parameters, so we used them!

Parameters of the model

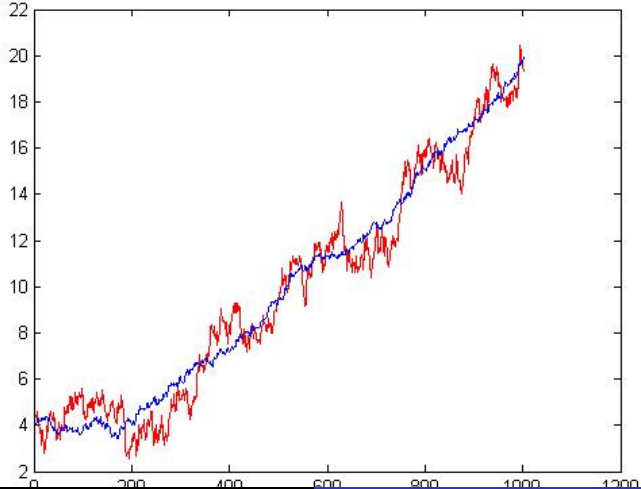
Table: Estimated Parameters of the Two-Factor Model

Parameters	Sim. Mean	Sim. Std. Dev.	Real Est.
α	0.0295	0.0085	0.0262
σ	0.2588	0.0056	0.2581
μ	-0.0017	0	-0.0017
γ	1.3147	0	1.3147
τ	0.0764	0	0.0764

S_t simulated given from real L_t



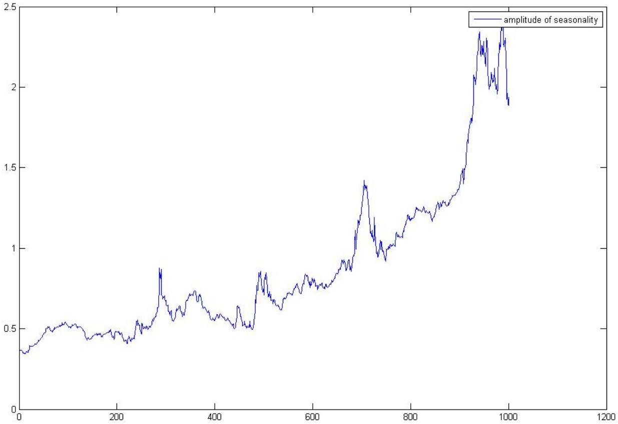
S_t From simulated L_t



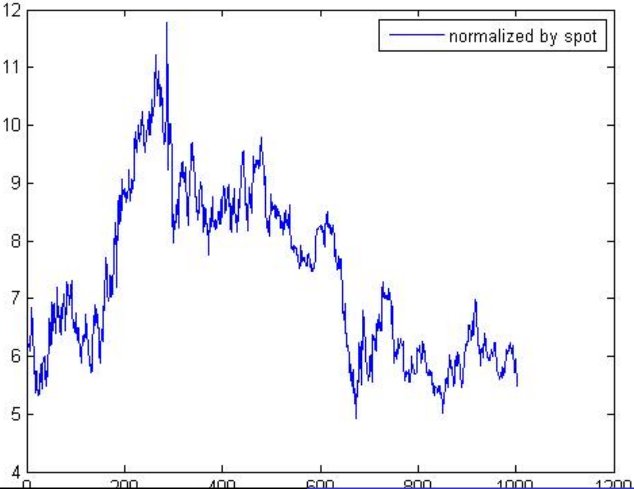
Modelling the seasonality term

The seasonality modelled, take distance of contracts in seasonality, since it is only the amplitude that keeps changing. Divide it by the spot price and we get the following process. Looks like a nice mean reverting model whose parameters can be captured by a mean reverting OU process, with some f(t) to be constant. Model an entire set of simulations.

Seasonality width



Seasonality width normalized by spot (width/spot)



Seasonality width normalized by spot (width/spot)

Use an OU process.
Can easily resimulate seasonality terms.

Models tested

$dS_t = -\lambda S_t dt + S_t^r dL_t$

Where L_t is an alpha stable levy process and an NIG process.

No time for this presentation to show empirical results.

Calibrated gas spot prices and seasonality using the above.

Calculated explicit futures price from the above equation.

Conclusions

Helps capture gas futures curve dynamics.

Infact turns out to be a three factor model, since seasonality dynamics are modelled seperately.

Could use improvements in better capturing the bootstrap error, as every cent counts in futures modelling!

References

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[4] C.R. Nelson and A.F. Siegel (1987), "Parsimonious Modeling of Yield Curves," Journal of Business, 60, 473-489.