Regime switching models for Natural gas

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Outline

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Outline

- Variations of the model
- Calibration to Futures, procedure and empirical results
- Calibration to Futures on options, procedure and empirical results
- Conclusion

Natural gas markets

- Natural gas is one of the cleanest, cheapest and most efficient sources of energy.
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- Alberta is home to a large natural gas resource base and accounts for just over 80 per cent of the natural gas produced in Canada.
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- 87 trillion cubic feet (Tcf) of recoverable, conventional natural gas is still beneath our feet.
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- Flammable gas, commonly used to fuel household appliances, Heating homes and businesses

Natural gas markets

- 75 per cent of the natural gas consumed in Alberta is used by the industrial sector (including electricity generation).
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- Displays seasonality, need models to capture it.
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- Natural gas futures and options on futures are sold in the market.
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- Natural gas storages used as a hedging instrument.
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Motivation of the paper

- One factor mean reverting models typically used in literature of natural gas storage evaluation.
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- Multi factor models are computationally expensive.
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- Introduces a one factor regime switching model that seems to work almost as well as 2 factor models with respect to fitting forward curves.

Motivation

- When regimes switch between MR and MR's equilibrium price, reproduces dynamics of Xu(2004)
- Regimes switch between GBM and GBM (different signs of drifts), reproduces Schwartz(1997)
- Our focus today is on its empirical results and not its implications in natural gas storage pricing.

Schwartz model

- Extends the typical mean reverting OU model.
- Adds additional stochastic factor of convenience yield.

Convenience Yield

- A factor implied by the futures or other derivative prices of commodities
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- Net benefit minus the cost of holding energy
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- Unlike financial derivatives, storage of energy products is costly.
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Convenience yield

- Physical ownership of commodity, carries an associated flow of services
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- Agent has the option of flexibility with regards to consumption, but this decision of postpone consumption implies storage expenses.
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- δ = benefit of direct access - cost of carry
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- Forward price is: $F(t,T) = S_t \mathbb{E}_Q \left[e^{\int_t^T (r_s - \delta_s) ds} \right].$
-

Schwartz model

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathbb{P})$ be a filtered probability space. As our state processes we consider the spot commodity asset S_t and the spot instantaneous convenience yield δ_t. According to Gibson and Schwartz, under the risk-neutral measure $\mathbb{Q}$,

$$
\begin{aligned}
dS_t &= (r_t - \delta_t) S_t \, dt + \sigma S_t \, dW_t^1 , \\
d\delta_t &= \kappa(\theta - \delta_t) dt + \gamma \, dW_t^2 ,
\end{aligned}
$$

with W^1, W^2 1-dimensional Wiener processes satisfying $d\langle W^1, W^2 \rangle_t = \rho \, dt$.

Xu (2004)

- Includes seasonality and mean reverting long run mean.

$$S_t = f(t) + X_t,$$

where

$$dX_t = \alpha(L_t - X_t)dt + \sigma(t)X_t dW_t^1$$

$$dL_t = \mu(\gamma - L_t)dt + \tau L_t dW_t^2,$$

$$\sigma(t) = \exp\left(c + \sum_{j=1}^{2} \lambda_j \cos 2\pi jt + \omega_j \sin 2\pi jt\right)$$

$$f(t) = bt + \sum_{j=1}^{2} \beta_j \cos 2\pi jt + \eta_j \sin 2\pi jt.$$

One factor mean reverting (MR) model (back to the paper)

2.1 One-factor mean-reverting model (MR model)

Let P denote the natural gas spot price. In the MR model, the gas spot price follows a mean-reverting stochastic process with the seasonality effect represented in the drift term. The risk adjusted gas spot price is modeled by a stochastic differential equation (SDE) given by

$$dP = \alpha(K_0 - P)dt + \sigma P dZ + S(t)Pdt \;, \tag{2.1}$$

$$S(t) = \beta_A \sin(2\pi(t - t_0 + C_A(t_0))) + \beta_{SA} \sin(4\pi(t - t_0 + C_{SA}(t_0))) \;, \tag{2.2}$$

where

α $\alpha > 0$ is the mean-reversion rate,

K_0 $K_0 > 0$ is the long-term equilibrium price,

MR model

σ $\sigma > 0$ is the volatility,

dZ is an increment of the standard Gauss-Wiener process,

$S(t)$ is a time-dependent term so that $S(t)Pdt$ is the price change at time t contributed by the seasonality effect. Note that multiplying $S(t)$ with P guarantees the price of natural gas always stays positive,

β_A is the annual seasonality parameter,

t_0 is a reference time satisfying $t_0 < t$.

MR model

$C_A(t_0)$ is the annual seasonality centering parameter for t_0. We define

$$C_A(t_0) = A_0 + D(t_0), \tag{2.3}$$

where A_0 is a constant time adjustment parameter obtained through calibration; $D(t_0)$ is the distance between the reference time t_0 and the first date in January in the year of t_0. Thus, by calibrating the value of A_0, we are able to determine the evolution of the annual seasonality effect over time.

β_{SA} is the semiannual seasonality parameter,

$C_{SA}(t_0)$ is the semiannual seasonality centering parameter for t_0. Similar to the definition of $C_A(t_0)$, we define

$$C_{SA}(t_0) = SA_0 + D(t_0), \tag{2.4}$$

where the constant time adjustment parameter SA_0 is obtained from a calibration process.

Effect of seasonality on gas dynamics

Remark 2.1 (Effect of the seasonality term on gas price dynamics). *We can rewrite equation (2.1) as*

$$dP = \alpha K_0 dt + (S(t) - \alpha)Pdt + \sigma PdZ. \tag{2.5}$$

Since $-(|\beta_A| + |\beta_{SA}|) \leq S(t) \leq |\beta_A| + |\beta_{SA}|$ *according to equation (2.2), if*

$$|\beta_A| + |\beta_{SA}| > \alpha, \tag{2.6}$$

then there exists certain periods of time within which $S(t) - \alpha > 0$*. In this case, if* P *is large and* $(S(t) - \alpha)Pdt \gg \alpha K_0 dt$ *in equation (2.5), then the process (2.1) becomes a GBM process with positive drift rate due to the strong seasonality effect. At other times, the process is mean-reverting. Note that the deseasoned process (i.e., setting* $S(t) = 0$ *in SDE (2.1)) is a mean-reverting process. As indicated in our calibration results in Section 3.1.4, condition (2.6) is typically satisfied by the calibrated parameters.*

Regime switching model

- Previous model is our MR model and can be varied to GBM, varying parameters.
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- The model to be proposed has two regimes, switches between a combination of the above.
-
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Regime switching

The switch between two regimes can be modeled by a two-state continuous-time Markov chain $m(t)$, taking two values 0 or 1. The value of $m(t)$ indicates the regime in which the risk adjusted gas spot price resides at time t. Let $\lambda^{0\to 1}dt$ denote the probability of shifting from regime 0 to regime 1 over a small time interval dt, and let $\lambda^{1\to 0}dt$ be the probability of switching from regime 1 to regime 0 over dt. Then $m(t)$ can be represented by

$$dm(t) = (1 - m(t-))dq^{0\to 1} - m(t-)dq^{1\to 0}, \tag{2.7}$$

where $t-$ is the time infinitesimally before t, and $q^{0\to 1}$ and $q^{1\to 0}$ are the independent Poisson processes with intensity $\lambda^{0\to 1}$ and $\lambda^{1\to 0}$, respectively.

Regime switching

In the regime-switching model, the risk adjusted natural gas spot price is modeled by an SDE given by

$$dP = \alpha^{m(t-)}(K_0^{m(t-)} - P)dt + \sigma^{m(t-)}PdZ + S^{m(t-)}(t)Pdt, \tag{2.8}$$

$$S^{m(t-)}(t) = \beta_A^{m(t-)}\sin(2\pi(t - t_0 + C_A(t_0))) + \beta_{SA}^{m(t-)}\sin(4\pi(t - t_0 + C_{SA}(t_0))) \,. \tag{2.9}$$

As indicated in equations (2.8-2.9), within a regime $k \equiv m(t-)$ the gas spot price follows the process (2.1-2.2) with parameters $\alpha^k, K_0^k, S^k(t), \sigma^k$ (but the signs of α^k and K_0^k are not constrained). Meanwhile, the stochastic factors for the two regimes are perfectly correlated. Note that we assume that the centering parameters $C_A(t_0)$ and $C_{SA}(t_0)$, as given in equations (2.3) and (2.4), respectively, are identical for two regimes in order to reduce the number of calibrated parameters.

Regime switching

Remark 2.2 (Mean-reverting or GBM-like process). *From the model (2.8-2.9), the* deseasoned *spot price in regime $m(t-)$ can follow either a mean-reverting process or a GBM-like process by setting parameter values.*

If we choose $\alpha^{m(t-)} > 0$ and $K_0^{m(t-)} > 0$, then the deseasoned gas price (obtained from setting the seasonality term $S^{m(t-)}(t) = 0$ in SDE (2.8)) follows a mean-reverting process

$$dP = \alpha^{m(t-)}(K_0^{m(t-)} - P)dt + \sigma^{m(t-)}PdZ \tag{2.10}$$

with equilibrium level $K_0^{m(t-)}$ and mean-reversion rate $\alpha^{m(t-)}$.

Regime switching

If we set $K_0^{m(t-)} = 0$ *in equation (2.8), then the deseasoned gas price SDE becomes*

$$dP = -\alpha^{m(t-)} P dt + \sigma^{m(t-)} P dZ. \qquad (2.11)$$

This is a GBM-like process. Specifically, if the drift coefficient $-\alpha^{m(t-)} > 0$, *then SDE (2.11) is a standard GBM process, i.e., gas price P will drift up at a rate* $|\alpha^{m(t-)}|$ *at time t; if* $-\alpha^{m(t-)} < 0$, *then the gas price will drift down at a rate* $|\alpha^{m(t-)}|$.

Variations of the regime switching model

MRMR variation

The processes in both regimes are mean-reverting with different equilibrium levels, i.e., $K_0^k > 0$, $\alpha^k > 0$, $k \in \{0, 1\}$ in SDE (2.8). In this variation, the equilibrium level of the gas spot price switches between two constants, K_0^0, K_0^1, which thus creates a sort of mean-reverting effect on the equilibrium level. This simulates the behavior of the equilibrium price in the two-factor model proposed by Xu (2004), where the gas spot price P follows a one-factor mean-reverting process and its equilibrium price evolves over time according to the other one-factor mean-reverting process.

Variations of the regime switching model

MRGBM variation

The process in one regime is mean-reverting while the other regime is a GBM process with a positive drift, i.e., $K_0^0 > 0, K_0^1 = 0, \alpha^0 > 0, \alpha^1 < 0$ in SDE (2.8). The mean-reverting regime represents the normal price dynamics, and the GBM regime can be regarded as the sudden drifting up of the gas price driven by exogenous events.

$$dP = \alpha^{m(t-)}(K_0^{m(t-)} - P)dt + \sigma^{m(t-)}PdZ + S^{m(t-)}(t)Pdt,$$

Variations of the regime switching model

GBMGBM variation

The processes in both regimes are GBM processes with a positive drift in one regime and a negative drift in the other, i.e., $K_0^0 = K_0^1 = 0, \alpha^0 < 0, \alpha^1 > 0$ in SDE (2.8). This simulates the behavior of the two-factor model in Schwartz (1997), where the risk adjusted commodity spot price process is modeled by a GBM-like process given by

$$dP = (r - \delta)Pdt + \sigma PdZ. \tag{2.12}$$

Calibration to futures

3.1.1 Futures Price Valuation

Let $F^k(P,t,T)$ denote the natural gas futures price in regime k, $k \in \{0,1\}$, at time t with delivery at T, while the gas spot price resides at P. Assuming the risk adjusted natural gas spot price follows the regime-switching model (2.8-2.9), we can write $F^k(P,t,T)$ as the risk neutral expectation of the spot price at T

$$F^k(p,t,T) = E^{\mathbf{Q}}[P(T) \mid P(t) = p, m(t) = k], \tag{3.1}$$

where $m(t)$ is the two-state Markov chain given in (2.7), representing the regime in which the risk adjusted gas spot price resides at t. From equation (3.1), F^k satisfies two PDEs that are coupled with each other given by

$$F_t^k + [\alpha^k(K_0^k - P) + S^k(t)P]F_P^k + \frac{1}{2}(\sigma^k)^2P^2F_{PP}^k + \lambda^{k\to(1-k)}(F^{1-k} - F^k) = 0\ ,\ \ k \in \{0,1\} \tag{3.2}$$

with the boundary conditions

$$F^k(P,T,T) = P\ ,\ \ k \in \{0,1\}. \tag{3.3}$$

Calibration to futures

The solution to PDEs (3.2) has the form

$$F^k(P,t,T) = a^k(t,T) + b^k(t,T)P, \tag{3.4}$$

where functions a, b are independent of P. Substituting equation (3.4) into equations (3.2-3.3) gives an ODE system

$$\begin{aligned} a_t^k + \lambda^{k\to(1-k)}(a^{1-k} - a^k) + \alpha^k K_0^k b^k &= 0 \\ b_t^k - \left[\alpha^k - S^k(t) + \lambda^{k\to(1-k)}\right] b^k + \lambda^{k\to(1-k)} b^{1-k} &= 0 \ , \ \ k \in \{0,1\} \end{aligned} \tag{3.5}$$

subject to the boundary conditions

$$a^k(T,T) = 0 \ \ ; \ \ b^k(T,T) = 1 \ , \ \ k \in \{0,1\}. \tag{3.6}$$

Calibration to futures

Remark 3.1. *For the regime-switching model, equation (3.4) and the ODE system (3.5-3.6) imply that the futures prices* $F^k(t,T), k \in \{0,1\}$ *at time* t *when the gas spot price is known are independent of the spot price volatilities* σ^0, σ^1*. Similar observations indicate that the futures price is independent of spot price volatility for the MR model. Consequently, the volatility needs to be calibrated using financial instruments other than futures contracts; in this paper, we choose options on futures (see Section 3.2 for the detailed calibration procedure).*

Calibration to futures

- So options on futures is used to find the volatility parameter, rest of the parameters can be found from the futures contracts!

Data

- Data set contains 51 observations in 51 months. From Feb 2003 to July 2007
- Each observation contains delivery prices for the first 14 contracts that correspond to the delivies in the next 14 consecutive months starting from the month of observation.
- For calibration, we need gas spot prices, there exists a gas spot market in Henry hub but we do not use it.
- Delivery period of contracts in spot and futures is different.

Data

- Spot market – delivery lasts for only 24 hours
- Futures market – delivery over a whole month
- So instead of using the spot price, at any point of time we use the next months futures contract delivery price instead.
- For calibration, there are now a total of 13*51 = 663 futures prices.

Calibration procedure

Let $\theta = \{\alpha^k, K_0^k, \beta_A^k, \beta_{SA}^k, A_0, SA_0, \lambda^{k \to (1-k)} | k \in \{0,1\}\}$ denote the set of parameters we need to obtain through calibrating to the futures price data.

$$\min_\theta \sum_t \sum_T \left(\hat{F}^{\hat{k}(t;\theta)}(P(t), t, T; \theta) - F(t,T) \right)^2 \text{ , where}$$

$$\hat{k}(t;\theta) = \arg\min_{k \in \{0,1\}} \sum_T \left(\hat{F}^k(P(t), t, T; \theta) - F(t,T) \right)^2 ,$$

where $F(t,T)$ is the market futures price on the observation day t with maturity T; $\hat{F}^{\hat{k}(t;\theta)}(P(t), t, T; \theta)$ is the corresponding model implied futures price in regime $\hat{k}(t;\theta)$ calculated numerically from equations (3.4-3.6) using the market spot price $P(t)$ and the parameter set θ. In equation (3.7), the range of t consists of all the observation days in our sample data and that of T covers the thirteen consecutive delivery months starting two months after the month of t.

Calibration

- Note:

Our calibration results are sensitive to the starting values used in the optimization procedure. For example, if the initial estimates for the parameters has either the MRMR or MRGBM form, the calibrated parameters retain the same form. As we shall see below, good fits to the data can be obtained with either MRMR or MRGBM. However, if we use initial parameters consistent with GBMGBM, then the optimization procedure converges to the MRGBM parameters.

This indicates that the MRMR or MRGBM models are consistent with the market data, while the GBMGBM model does not appear to be consistent with market data.

Calibration result from futures contract

		MR	MRMR	MRGBM
Parameter	Description	Estimate	Estimate	Estimate
α (α^0)	Mean-reversion rate (for regime 0)	0.406	0.430	0.435
K_0 (K_0^0)	Equilibrium price (for regime 0)	8.678	4.466	4.748
β_A (β_A^0)	Annual seasonality parameter (for regime 0)	0.527	0.600	0.550
β_{SA} (β_{SA}^0)	Semiannual seasonality parameter (for regime 0)	0	0	0
A_0	Annual seasonality time adjustment parameter	0.483	0.441	0.457
α^1	Mean-reversion rate for regime 1		1.033	−0.650
K_0^1	Equilibrium price for regime 1		11.709	0
β_A^1	Annual seasonality parameter for regime 1		0.571	0.555
β_{SA}^1	Semiannual seasonality parameter for regime 1		0	0
$\lambda^{0\to1}$	Intensity of the jump from regime 0 to regime 1		0.304	0.283
$\lambda^{1\to0}$	Intensity of the jump from regime 1 to regime 0		0.975	2.290

TABLE 3.1: *Estimated parameter values for the three models using 663 monthly observed futures price data from February 2003 to July 2007. The column MR represents the MR model. The columns MRMR and MRGBM represent the MRMR and MRGBM variation of the regime-switching model, respectively.*

Regime state

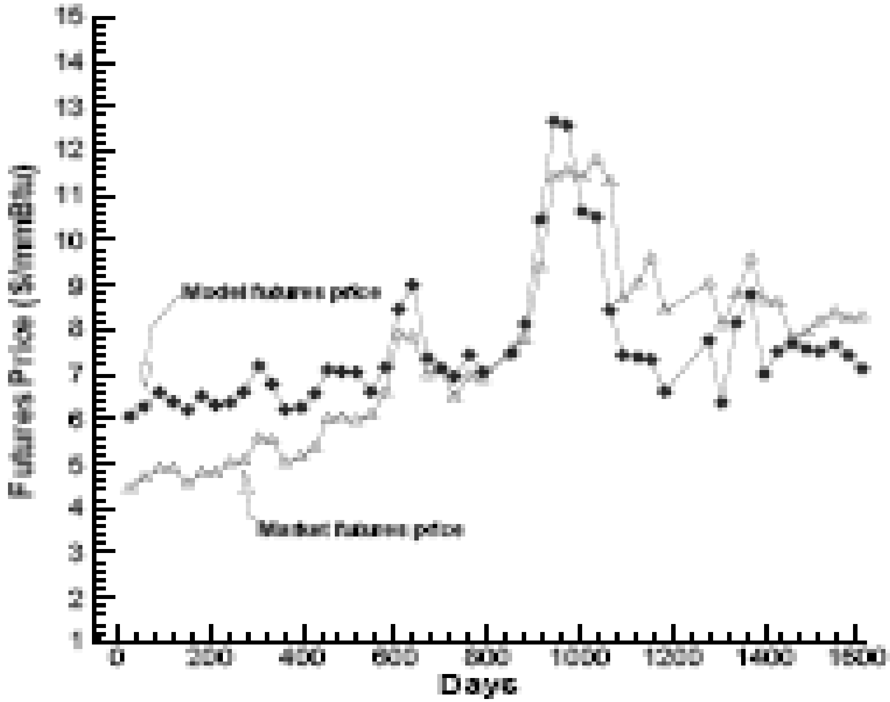
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2003	N/A	0	0	0	0	0	0	0	0	0	0	0
2004	0	0	0	0	0	0	0	1	1	0	0	1
2005	1	1	0	0	N/A	0	0	0	0	0	0	0
2006	1	1	1	1	1	N/A	N/A	1	1	1	1	1
2007	1	1	1	1	1	1	1	N/A	N/A	N/A	N/A	N/A

TABLE 3.2: *Regimes where the realized market gas spot price resides at various times, where the spot price follows the MRGBM variation of the regime-switching model. The Table shows that 29 months correspond to regime 0 and 22 months correspond to regime 1. The N/A in the table corresponds to missing data.*

Error for forward prices

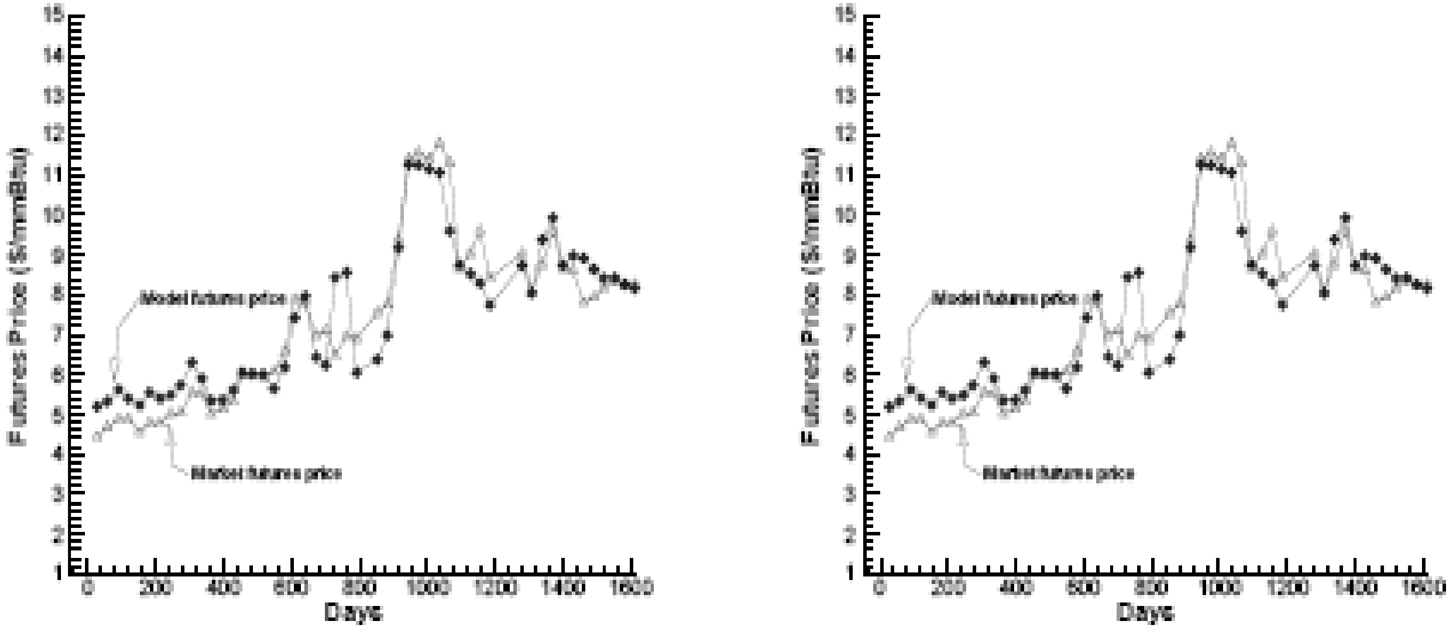
Contract maturity	MR	MRMR	MRGBM	MR	MRMR	MRGBM
	Mean absolute error					
	In Dollars			In Percentage		
Month+2	0.278	0.240	0.248	3.98	3.57	3.64
Month+3	0.499	0.386	0.388	6.99	5.70	5.56
Month+4	0.645	0.487	0.470	8.96	7.01	6.57
Month+5	0.684	0.504	0.471	9.44	7.08	6.45
Month+6	0.741	0.486	0.502	10.13	6.60	6.60
Month+7	0.827	0.493	0.528	11.14	6.59	6.75
Month+8	0.872	0.492	0.548	11.86	6.66	7.11
Month+9	0.949	0.505	0.563	12.97	6.92	7.37
Month+10	1.011	0.557	0.574	13.93	7.61	7.53
Month+11	1.037	0.603	0.622	14.62	8.20	8.20
Month+12	1.075	0.580	0.640	15.60	8.21	8.53
Month+13	1.118	0.580	0.677	16.65	8.53	9.21
Month+14	1.152	0.585	0.698	17.68	8.97	9.74
Overall	0.838	0.500	0.533	11.84	7.05	7.17

Forward curve fitting



(A) MR

Forward curve fitting



(B) MRMR (C) MRGBM

FIGURE 3.1: *Comparison between the model and the market futures prices for the contract with the longest maturity (for the delivery after 14 months) in the sample across all observation days starting from February 2003. The x-axis represents the number of days between the observation day and the starting date. The model implied prices are computed using the calibrated parameters in Table 3.1. MR represents the MR model. MRMR and MRGBM represent the MRMR and MRGBM variation of the regime-switching model, respectively.*

Futures Option Valuation

Let $\tilde{V}^k(F,t,T_v)$ denote the European call option value in regime k at time t with maturity at T_v, where F represents the price of the underlying futures contract at time t. Let K denote the strike price of the option. Let $F^k(t,T)$ represent the price of the underlying futures contract in regime k at time t with maturity at T, where T satisfies $T \geq T_v$.

Futures option evaluation

In NYMEX, the trading of a European option ends on the business day immediately preceding the expiration of the underlying futures contract. As a result, we can assume $T_v = T$, and we will thus use T as the maturity for both an option and its underlying futures contract.

We can write $\bar{V}^k(F,t,T)$ in the form of the risk neutral expectation as

$$\begin{aligned} \bar{V}^k(f,t,T) &= e^{-r(T-t)} E^Q \left[\left(F^{m(T)}(T,T) - K \right) 1_{F^{m(T)}(T,T) \geq K} \mid F^k(t,T) = f, m(t) = k \right] \\ &= e^{-r(T-t)} E^Q \left[\left(P(T) - K \right) 1_{P(T) \geq K} \mid a^k(t,T) + b^k(t,T) P(t) = f, m(t) = k \right], \end{aligned} \tag{3.8}$$

where $1_{x \geq y}$ is an indicator function that returns 1 if $x \geq y$, or 0 if $x < y$; the second equality above uses the fact that $F^{m(T)}(T,T) = P(T)$ at maturity T as well as the relation (3.4) between futures price F and spot price P at time t assuming the risk neutral gas spot price follows the regime-switching model (2.8-2.9). Let $V^k(P,t,T)$ represent a synthetic European call option on spot price P at time t, in regime k with maturity T. Then we can write $V^k(P,t,T)$ in the form of the risk neutral expectation as

$$V^k(p,t,T) = e^{-r(T-t)} E^Q \left[\left(P(T) - K \right) 1_{P(T) \geq K} \mid P(t) = p, m(t) = k \right]. \tag{3.9}$$

Futures option evaluation

$$V^k(p,t,T) = e^{-r(T-t)}E^{\mathbb{Q}}\left[(P(T)-K)1_{P(T)\geq K} \mid P(t)=p, m(t)=k\right]. \qquad (3.9)$$

Comparing equations (3.8) and (3.9), we have

$$V^k\left(\frac{f-a^k(t,T)}{b^k(t,T)}, t, T\right) = \bar{V}^k(f,t,T), \qquad (3.10)$$

where a^k and b^k are computed from the ODE system (3.5-3.6). As a result, we can compute $\bar{V}^k(F,t,T)$ using equation (3.10) as long as we are able to solve for $V^k(P,t,T)$.

Let r denote the constant riskless interest rate. Assuming that the spot price process follows SDE (2.8-2.9) and using the risk neutral expectation formulation (3.9), we find that the synthetic option value V^k satisfies the coupled PDEs

$$\begin{aligned} &V_t^k + \left[\alpha^k(K_0^k - P) + S^k(t)P\right]V_P^k + \frac{1}{2}(\sigma^k)^2P^2V_{PP}^k - rV^k + \\ &\lambda^{k\to(1-k)}(V^{1-k} - V^k) = 0\ ,\ k\in\{0,1\} \end{aligned} \qquad (3.11)$$

subject to the boundary conditions

$$V^k(P,T,T) = \max[P-K,0]\ ,\ k\in\{0,1\}, \qquad (3.12)$$

Futures option

We will solve equations (3.11-3.12) numerically in the computational domain $P \in [0, P_{\max}]$ with $P_{\max} \gg K$. For this purpose, we need to impose boundary conditions at the computational boundary $P = 0$ and $P = P_{\max}$. At the $P = 0$ boundary, taking the limit as $P \to 0$, equations (3.11) become

$$V_t^k + \alpha^k K_0^k V_P^k - rV^k + \lambda^{k \to (1-k)}(V^{1-k} - V^k) = 0\ ,\ k \in \{0,1\}. \tag{3.13}$$

computational domain. At the $P = P_{\max}$ boundary, we make the assumption that $V^k(P_{\max}, t, T) \to$ payoff. In other words, we impose the Dirichlet boundary condition

$$V^k(P_{\max}, t, T) = P_{\max} - K\ ,\ k \in \{0,1\}. \tag{3.14}$$

Calibration

$$\min_{\sigma^0,\sigma^1} \sum_K \left(V^k(F(t,T_1),t,T_1;\theta,K,\sigma^0,\sigma^1) - C(t,T_1;K)\right)^2,$$

Thank you!