Using Evolutionary Algorithms to Estimate Jump-Diffusion Models

John Sheriff

February 26, 2009

Motivation

- Many problems involve optimizing some complex function, often over a high dimensional space.
- Evolutionary, or genetic, algorithms may provide an efficient, flexible, robust approach to such problems.

Evolutionary Algorithm

Evolutionary algorithms are designed search possible solutions in some probabilistic fashion to find global optima, by iteratively updating a population of candidate solutions, in some Darwinian manner.

General references in the area include

- Bäck, T., Evolutionary Algorithms in Theory and Practice
- Holland, J.H., Adaptation in natural and artificial systems

Basic Structure

1. Initialize the population
2. Randomly vary individuals (mutation, recombination)
3. Evaluate “fitness” of individuals
4. Select new population
5. Repeat 2-4

Some Examples from Finance

- Portfolio management/trading
  - L'Her, Mouakhar, Roberge, *Journal of Portfolio Management*, 2007
  - Allen, Karjalainen, *Journal of Financial Economics*, 1999
- Option pricing
  - Cont, Hamida, *Journal of Computational Finance*, 2005
  - Grace, *Applied Economic Letters*, 2000

Model Calibration Problem

Let $H = H(T)$ be payoff of some contingent claim at time T.

Let $\bar{V}(H)$ represent the observed price of this claim.

Assume $\bar{V}_i(T_i, K_i) = \bar{V}(H(T_i, K_i)),\ i = 1, \ldots, m$ to be a collection of observed prices on options having maturity T_i and strike K_i.

Model Calibration Problem con't

Given some market model, depending upon parameter θ, want to find measure $\mathbb{Q}(\theta)$, such that

$$\bar{V}_i = V_i(T_i, K_i; \theta),\ i = 1, \ldots, m$$

where $V_i(T_i, K_i; \theta) = e^{-rT_i} E^{\mathbb{Q}}[H(T_i, K_i)]$

Objective Function

Given market data and market model (parameterized by θ), the task is typically

$$\min_{\theta} G(\theta) = \sum_{i=1}^{n} w_i |V_i(T_i, K_i; \theta) - \bar{V}_i|^2$$

Optimizing $G(\theta)$ may be problematic since:

- may not be convex
- may not have unique solution
- gradient methods may be challenging

Regularization Approach

One way to address some of these problems is to attach a penalty function

$$\min_{\theta} G(\theta) + \alpha F(\theta)$$

Examples of penalty function F:

- relative entropy
  - Cont, Tankov, *Journal of Computational Finance*, 2004
- smoothness
  - Jackson, Suli, Howison, *Journal of Computational Finance*, 1999

Evolutionary Algorithm Approach

Optimize $G(\theta)$ directly

- convexity not a problem
- will deal with multiplicity of solutions
- avoids gradient methods
- avoids potential trade-off between calibration and penalty term

“Implementation”

Stock price model: $S_t = S_0 e^{X_t}$, where

$$X_t = \gamma t + \sigma W_t + \sum_{i=1}^{N_t} Y_i$$

$Y_i \sim N(\mu, \delta^2)$

$N_t \sim \text{Poisson}(\lambda t)$

σ constant

Initialize population

Generate $\{\theta_0^i\}_{i=1}^N = \{\gamma_0^i, \mu_0^i, \delta_0^i, \lambda_0^i\}_{i=1}^N$ from some prior distribution

Option 1:
    Sample randomly over some plausible set

Option 2:
    Use historical data to direct initialization

“Fitness” of individual and selection criterion

At the k^{th} iteration, fitness of $\theta_k^i, \ i = 1, \ldots, N$ is given by $G(\theta_k^i)$

Selection of individuals for recombination and for inclusion in the next generation occurs with probability

$$\frac{\exp\left[-\beta_k G(\theta_k^i)\right]}{\sum_{j=1}^{N} \exp\left[-\beta_k G(\theta_k^j)\right]}$$

β_k represents selection pressure.

Mutation

Let A represent the covariance matrix for parameters γ, μ, δ, λ.
For each $\theta_k^i,\ i = 1, \ldots, N$,

$$\tilde{\theta}_k^i = \theta_k^i + B\epsilon^i,$$

where $A = BB'$ and ϵ^i is a standard normal vector.

Recombination

Pairs of individuals $(\tilde{\theta}_k^i, \tilde{\theta}_k^j)$ chosen from mutated population.
Selected with probability proportional to fitness.
New individual created

$$\tilde{\theta}_k^r = \alpha\tilde{\theta}_k^i + (1-\alpha)\tilde{\theta}_k^j,$$

and added to population.

Selection

Mutation and recombination produces $\{\tilde{\theta}_k^i\}_{i=1}^R,\ R > N$.

Select N individuals to form next generation, $\{\theta_{k+1}^i\}_{i=1}^N$

Each individual $\tilde{\theta}_k^i$ is selected with probability

$$\frac{\exp\left[-\beta_k G(\tilde{\theta}_k^i)\right]}{\sum_{j=1}^N \exp\left[-\beta_k G(\tilde{\theta}_k^j)\right]}$$

Stopping and Convergence

Stopping:

- Fixed number of generations/run time
- Improvement in fitness of fittest member plateaus
- Difference in fitness of most and least fit

Convergence:

- algorithm will converge under fairly reasonable conditions
  - Cerf, R, Advances in Applied Probability, 1998
  - Del Moral, Miclo, Theoretical Aspects of Evolutionary Computing, 2001
  - Del Moral, Miclo, Communications in Mathematical Physics, 2003
- rate

Interpreting results

Algorithm produces population of candidate solutions with similar fitness.

What do solutions have to say about

- model
- pricing of other related claims

Last word